ELSEVIER
Journal of International Economics 62 (2004) 83–106
Journal of INTERNATIONAL ECONOMICS
www.elsevier.com/locate/econbase

Real GDP, real domestic income, and terms-of-trade changes ☆

Ulrich Kohli *

Swiss National Bank, Börsenstrasse 15, P.O. Box 2800, CH-8022 Zurich, Switzerland

Received 12 December 2001; received in revised form 10 June 2003; accepted 7 July 2003

Abstract

Real GDP tends to underestimate the increase in real domestic income and welfare when the terms of trade improve. An improvement in the terms of trade is similar to a technological progress, but when computing real GDP, the national accounts treat the former as a price phenomenon and the latter as a real event. Calculations for 26 countries show that the divergence can add up to more than 10% of GDP in less than two decades. Our analysis has a solid theoretical foundation, being based on the GNP/GDP function approach to modeling the production sector of an open economy.

Keywords: Real GDP; GDP deflator; Terms of trade; Real income; Economic growth

JEL classification: O11; O41; C43; F11

1. Introduction

The economic performance of Switzerland over the long run is paradoxical. In most international comparisons, Switzerland is found to have a growth rate that is significantly lower than that of other industrialized nations. And yet, in terms of average living standards, Switzerland always ranks among the top nations. How can Switzerland go slower than everybody else, and nonetheless stay ahead?

☆ Earlier versions of this paper were presented at the Economic Measurement Group (EMG) Workshop, University of New South Wales, Sydney, at the Hong Kong University of Science and Technology, and at the 2003 annual meetings of the Canadian Economics Association, Ottawa.
* Tel.: +41-1-631-3233/34; fax: +41-1-631-3188.
E-mail address: Ulrich.Kohli@snb.ch (U. Kohli).
URL: http://www.unige.ch/ses/ecopo/kohli/kohli.html.

0022-1996/$ - see front matter
doi:10.1016/j.jinteco.2003.07.002

For the period 1980–1996, for instance, Switzerland, with an average real GDP growth rate of 1.3%, occupies the last position in a sample of 26 OECD countries. One could of course argue that this is a sign of convergence. If Switzerland has a relatively high living standard initially, it is perfectly possible that it grows less rapidly than its neighbors, and nevertheless that it maintain its lead position for a while yet. Sooner or later, though, it will be caught up. It turns out, however, that the Swiss growth paradox is not new. According to Dewald's (2002) data that span the period 1880–1995, Switzerland occupies the second-last position in a sample of 12 countries in terms of per-capita real growth. Knowing that 19th century Switzerland was a poor country in European comparison, how can one explain that it is today one of the countries where real income is highest?

The answer to this puzzle has to do, at least partially, with the improvements in the terms of trade that Switzerland has enjoyed over time. From 1980 to 1996, for instance, Switzerland's terms of trade have improved by a stunning 34%. In many ways, an improvement in the terms of trade is similar to a technological progress. It means that, for a given trade-balance position, the country can either import more for what it exports, or export less for what it imports. Put simply, it makes it possible to get more for less. An improvement in the terms of trade unambiguously increases real income and welfare. Yet, unlike a technological progress, the beneficial effect of an improvement in the terms of trade is not captured by real GDP, which focuses on production per se. In fact, if real GDP is measured by a Laspeyres quantity index, as it is still the case in most countries, an improvement in the terms of trade will actually lead to a *fall* in real GDP.

Real GDP is often used as a proxy of a country's real income, even though official statisticians warn against such a practice.[1] Thus, Prescott (2002), who singles out Switzerland for its poor economic performance over the past three decades, focuses exclusively on real GDP. We argue in this paper that real GDP can be a very misleading indicator of a country's welfare in the face of changing terms of trade. It is therefore important to distinguish between real GDP, on one hand, and real domestic income, on the other. Real GDP focuses on production possibilities, whereas real income stresses consumption (or more generally absorption) possibilities and, ultimately, welfare.[2] We show that real GDP systematically underestimates growth in real income when the terms of trade improve. The distinction between real GDP and real income implies differences between the corresponding price indexes. The implicit GDP price deflator, which is obtained by dividing nominal GDP by real GDP, will point at higher inflation than the income price deflator when the terms of trade improve. In fact, it turns out that a drop in the price of imports, holding all other prices constant, leads to an *increase* in the GDP price deflator.

[1] See United Nations (2002), Section 16.K, for instance.

[2] Real income and welfare are clearly very different concepts, but the fact remains that an increase in real income will, other things equal, allow for an increase in welfare.

2. Preliminary analysis

The difference between real GDP and real domestic income can be illustrated in the familiar two-endproducts model of international trade theory. Let the quantities produced at time t be denoted by $y_{i,t}$, the quantities consumed by $q_{i,t}$ and their prices by $p_{i,t}$, $i \in \{1, 2\}$. *Nominal* GDP (π_t) can be thought of as the value of domestic production. It can be expressed as:

$$\pi_t \equiv p_{1,t} y_{1,t} + p_{2,t} y_{2,t}. \tag{1}$$

Ignoring indirect taxes and subsidies, nominal GDP can also be thought of as nominal domestic income.[3] If, moreover, trade is balanced, domestic income equals domestic expenditures, and we have:

$$\pi_t = p_1 q_{1,t} + p_{2,t} q_{2,t}. \tag{2}$$

Real GDP is conventionally measured by a direct, base-weighted Laspeyres quantity index ($Y^L_{t,0}$) relative to the base period (period 0).[4] Assuming that base-period prices are set to unity ($p_{i,0} = 1$), we get:

$$Y^L_{t,0} \equiv \frac{y_{1,t} + y_{2,t}}{y_{1,0} + y_{2,0}}. \tag{3}$$

Let $\Pi_{t,0}$ be the index of nominal GDP:

$$\prod_{t,0} \equiv \frac{\pi_t}{\pi_0} = \frac{p_{1,t} y_{1,t} + p_{2,t} y_{2,t}}{y_{1,0} + y_{2,0}}. \tag{4}$$

The GDP implicit price index ($P^P_{t,0}$) can then be obtained by deflation:

$$P^P_{t,0} \equiv \frac{\prod_{t,0}}{Y^L_{t,0}} = \frac{p_{1,t} y_{1,t} + p_{2,t} y_{2,t}}{y_{1,t} + y_{2,t}} = \frac{1}{s_{1,t} p^{-1}_{1,t} + s_{2,t} p^{-1}_{2,t}}, \tag{5}$$

where $s_{i,t} \equiv p_{i,t} y_{i,t} / \pi_t$ is the share of good i in production. Expression (5) shows that the traditional GDP implicit price deflator, being a current-weighted harmonic mean, has the Paasche form.

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[3] For simplicity, we ignore the foreign ownership of domestic factors of production and national factors held abroad; that is, we do not distinguish between GNP and GDP, or between domestic and national income. We also ignore depreciation; we thus do not make a distinction between GDP and NDP, or between gross domestic income (GDI) and net domestic income. In what follows, we will use the terms "income" and "domestic income" interchangeably.

[4] The United States has recently switched to a chained Fisher measure of real GDP. Although the Fisher index is far superior to the Laspeyres index, and chained indexes are to be preferred to runs of direct indexes, this switch has no bearing on the point made in this paper; also see footnote 7.

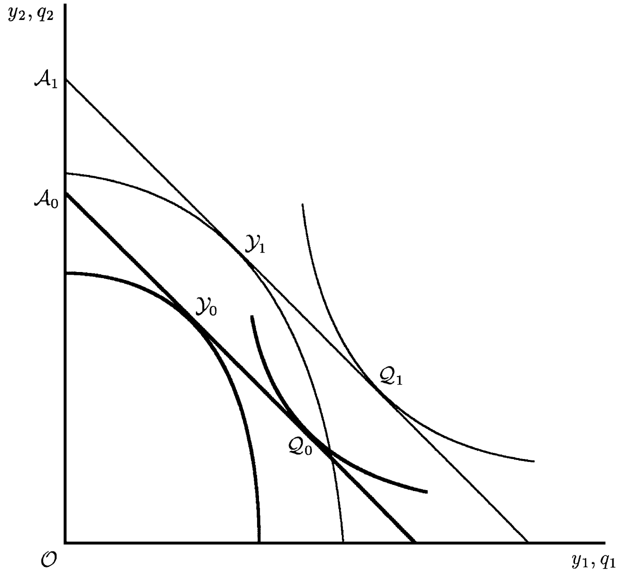

Fig. 1. Endproducts Model—A technological improvement increases real GDP and real income (from $\mathcal{OA}_0$ to $\mathcal{OA}_1$).

In the same vein, one can define a direct Laspeyres index of real domestic income or expenditures ($Q^L_{t,0}$) as:[5]

$$Q^L_{t,0} \equiv \frac{q_{1,t} + q_{2,t}}{q_{1,0} + q_{2,0}}, \tag{6}$$

with the corresponding implicit cost-of-living index ($C^P_{t,0}$):

$$C^P_{t,0} \equiv \frac{\prod_{t,0}}{Q^L_{t,0}} = \frac{p_{1,t}q_{1,t} + p_{2,t}q_{2,t}}{q_{1,t} + q_{2,t}} = \frac{1}{\omega_{1,t}p^{-1}_{1,t} + \omega_{2,t}p^{-1}_{2,t}}, \tag{7}$$

where $\omega_{i,t} \equiv p_{i,t}q_{i,t}/\pi_t$ is the expenditure share of good i.

We show in Fig. 1 the production possibilities frontier drawn for given domestic factor endowments and a given technology. Let the international price ratio be given by (minus) the slope of line $\mathcal{Y}_0\mathcal{Q}_0$.[6] Production takes place at point $\mathcal{Y}_0$. Under balanced trade consumption takes place at point $\mathcal{Q}_0$. The country is an importer of good 1 and an exporter of good 2.

Assume next a technological improvement that shifts the production possibilities frontier outwards. If all prices remain unchanged, production now takes place at $\mathcal{Y}_1$ and consumption at $\mathcal{Q}_1$. Real GDP and real domestic income clearly increase. Both the Laspeyres index of real GDP and the Laspeyres index of real domestic income are equal to the ratio $\mathcal{OA}_1/\mathcal{OA}_0$. Nominal GDP increases by the same factor. The GDP implicit price

5 We assume balanced trade for expository purposes only. This assumption will be relaxed later on.

6 This line is drawn with a unit slope since we have assumed that all prices are normalized to one in the base period as it is typically the case.

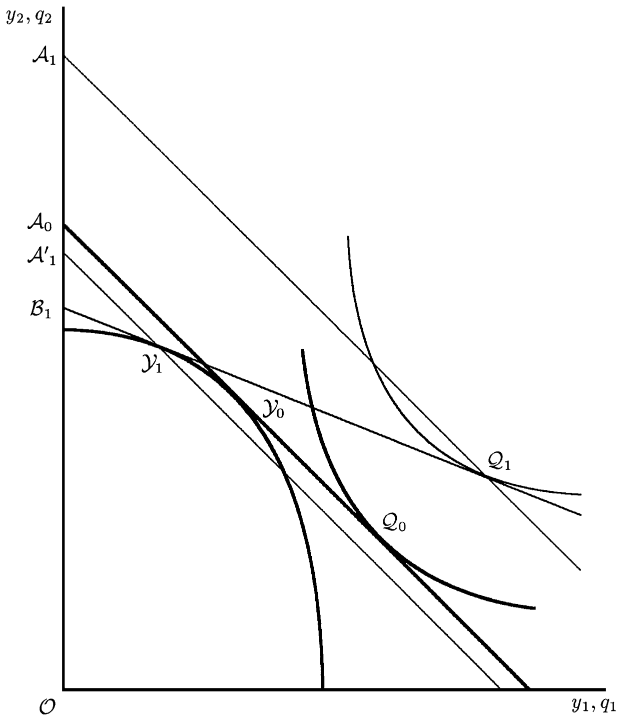

Fig. 2. Endproducts Model—An improvement in the terms of trade increases real income (from $\mathcal{OA}_0$ to $\mathcal{OA}_1$), but it reduces real GDP (from $\mathcal{OA}_0$ to $\mathcal{OA}'_1$).

deflator and the implicit cost-of-living index are both equal to one, which makes perfect sense since both disaggregate prices have remained unchanged by assumption. The increase in welfare made possible by the technological change is adequately reflected by the increase in production and in income.

Consider now Fig. 2 that shows the effect of an improvement in the terms of trade. Let us assume that it comes about as the result of a fall in the price of importables; we thus can use the second good as the numeraire. The international price line moves from $\mathcal{Y}_0\mathcal{Q}_0$ to $\mathcal{Y}_1\mathcal{Q}_1$. Production shifts towards the northwest to $\mathcal{Y}_1$ and consumption increases to $\mathcal{Q}_1$, which lies on a higher indifference curve. Welfare clearly goes up. The Laspeyres index of real domestic income is equal to $\mathcal{OA}_1/\mathcal{OA}_0$, which is greater than one. This demonstrates the increase in real income that takes place. The Laspeyres index of real GDP, on the other hand, is equal to $\mathcal{OA}'_1/\mathcal{OA}_0$, which is less than one ($\mathcal{A}'_1$ is the intercept of a line with unit slope drawn through $\mathcal{Y}_1$). That is, real GDP *falls*, even though welfare unambiguously increases as the result of the improvement in the terms of trade.[7]

[7] The *drop* in real GDP is due to the fact that the Laspeyres index only provides a linear approximation to what is depicted here as a nonlinear production possibilities frontier. In particular, it is not due to the absence of chaining, since there are only two states in this example. If one used a quantity index that is exact for the production possibilities frontier (e.g. the Fisher index assuming that the production possibilities frontier is square-rooted quadratic), real GDP would be found to be unchanged. It would still fail to capture the increase in welfare, though.

Given that the price of the second good does not change, we can express the index of nominal GDP as $\mathcal{OB}_1/\mathcal{OA}_0$, which is less than one. That is, nominal GDP decreases as the result of the drop in the price of good 1. The GDP implicit price deflator is equal to $\mathcal{OB}_1/\mathcal{OA}'_1$, whereas the implicit cost-of-living index is equal to $\mathcal{OB}_1/\mathcal{OA}_1$. Both are less than unity, thus underscoring the drop in the price level, but the cost-of-living index clearly registers a much larger fall than the GDP price deflator. This is due to the fact that the drop in the price of importables is more heavily weighted in consumption than in production ($\omega_1 > s_1$).

3. Trade in middle products

The analysis of the previous section assumes that all trade is in end products. In reality, most international trade is in middle products, to use the terminology of Sanyal and Jones (1982). The bulk of trade consists of raw materials and intermediate goods, and even so-called *finished* imports are typically not ready to meet final demand. They must still go through a number of changes in the importing country, such as unloading, transporting, financing, insuring, repackaging, wholesaling, and retailing. During this process, they are combined with domestic factor services, so that a significant proportion of their final price tag is generally accounted for by domestic activities. This militates in favor of treating imports as an input to the technology. Similarly, exports are not ready to meet final demand either. They must still enter the foreign production sector once they have reached their destination. As such, exports are conceptually different from final outputs intended for domestic use. The treatment of traded goods as middle products is also consistent with the national accounts, which distinguish between goods produced for domestic use and actual imports and exports, rather than between importables and exportables.

The uneven effect of an improvement in the terms of trade on real GDP and real domestic income can also be analyzed in the context of a model that allows for trade in middle products or intermediate goods. Treating imports as an input to the technology blurs the distinction between technological progress and an improvement in the terms of trade, however. This is because a change in the terms of trade exerts its impact *during* the production process. Production involves the transformation of inputs into outputs. In a narrow sense, this transformation is physical, but even in a closed economy, it also takes place through trade. Domestic production typically involves specialization, but specialization is only feasible in conjunction with trade. International trade further increases the scope for specialization. An improvement in the terms of trade may enable a country to exploit its comparative advantages even more and it is little different from a technological progress that would incite the country to specialize further. In many cases, it might be impossible to tell apart the two phenomena. If the cost-insurance-freight (CIF) price of landed imports drops, is it because transportation costs have fallen as the result of a technological progress, or is it because the foreign free-on-board (FOB) price has decreased, thus signifying an improvement in the terms of trade? A change in relative prices might require a technological improvement before it can be taken advantage of, just like a technological innovation may only be exploitable after an improvement in the terms of trade has taken place. Yet, while the two phenomena are similar and intertwined, they

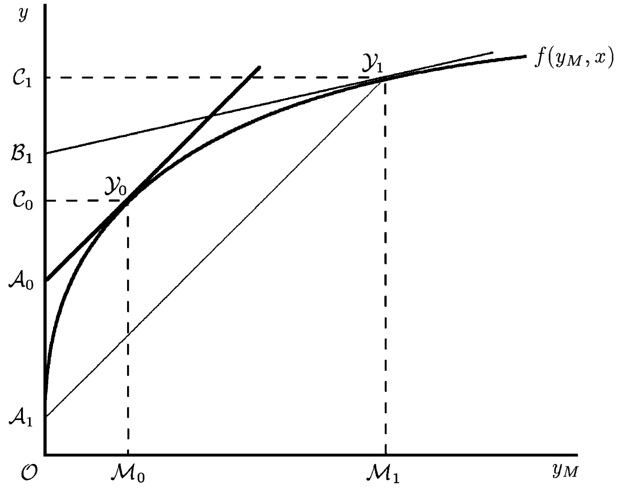

Fig. 3. Middle-Products Model—An improvement in the terms of trade increases real income (from $\mathcal{OA}_0$ to $\mathcal{OB}_1$), but it reduces real GDP (from $\mathcal{OA}_0$ to $\mathcal{OA}_1$).

are treated very differently by the national accounts, with technological progress being viewed as a real event and a change in the terms of trade as a price phenomenon.

For the remainder of this paper, imports are treated as inputs to the production process. Imports are combined with domestic factors (e.g. labor and capital) to produce one or more outputs, which can be absorbed at home or exported to the rest of the world. For simplicity, let us assume for the time being that all outputs, on one hand, and all domestic inputs, on the other, can be aggregated. The country's technology can be described by the following aggregate production function:

$$y_t = f(y_{M,t}, x_t), \tag{8}$$

where y_t is the quantity of aggregate output at time t, $y_{M,t}$ is the quantity of imports, and x_t is the quantity of the domestic composite factor. The production function is assumed to be increasing, linearly homogeneous, and quasi-concave. Let p_t and $p_{M,t}$ be the prices of output and imports, respectively. The terms of trade are given by the ratio $p_t/p_{M,t}$. Like in the previous section, factor endowments, the technology, and the terms of trade are taken as given. Optimization and perfect competition are assumed throughout.

The production function is shown in Fig. 3, with gross output as a function of the quantity of imports, for a given endowment of the domestic composite factor.[8] The slope of the production function is the marginal product of imports. Let the relative price of imports—the inverse of the terms of trade—be given by the slope of line $\mathcal{A}_0\mathcal{Y}_0$. This slope is unity since all prices are again normalized to unity for the base period. Profit maximization by producers will lead to an equilibrium at point $\mathcal{Y}_0$ where the marginal product of imports is equal to their marginal cost. The volume of imports is the distance

[8] See Kohli (1983) for additional details.

$\mathcal{O}\mathcal{M}_0$ and total output is equal to $\mathcal{O}\mathcal{C}_0$. If trade is balanced, exports are equal to $\mathcal{A}_0\mathcal{C}_0$, so that $\mathcal{O}\mathcal{A}_0$ is consumption of the final good.

Assume next that the terms of trade improve, as the result, for instance, of a drop in import prices. The terms of trade are now given by the slope of $\mathcal{B}_1\mathcal{Y}_1$, and equilibrium moves from point $\mathcal{Y}_0$ to point $\mathcal{Y}_1$. The country imports more. The marginal product of imports is lower, but their real price has fallen. Gross output is now equal to $\mathcal{O}\mathcal{C}_1$, and, with exports of $\mathcal{B}_1\mathcal{C}_1$ under balanced trade, consumption of the final good is $\mathcal{O}\mathcal{B}_1$.

In the context of this model that essentially treats imports as a negative output, expression (1) for nominal GDP (or nominal domestic income) must be adapted accordingly. Replacing $y_{1,t}$ by y_t and $y_{2,t}$ by $-y_{M,t}$, we get:

$$\pi_t \equiv p_t y_t - p_{M,t} y_{M,t}. \tag{9}$$

Nominal GDP clearly increases as the result of the drop in import prices: the index of nominal GDP is equal to $\mathcal{O}\mathcal{B}_1/\mathcal{O}\mathcal{A}_0$, which is greater than one. Since there is only one final good here, it is natural to identify its price (p) with the cost of living. Given that it remains unchanged by assumption, we can also interpret $\mathcal{O}\mathcal{B}_1/\mathcal{O}\mathcal{A}_0$ as the index of real income. The increase in real income clearly demonstrates the rise in welfare that takes place. The Laspeyres index of real GDP, on the other hand, is as follows:

$$Y^L_{t,0} \equiv \frac{y_t - y_{M,t}}{y_0 - y_{M,0}}. \tag{10}$$

In Fig. 3, this index is equal to $\mathcal{O}\mathcal{A}_1/\mathcal{O}\mathcal{A}_0$, which is less than one. That is, real GDP registers a drop, even though real income and welfare have unambiguously increased. This clearly shows the ambiguity of real GDP as a measure of a country's real income.[9]

The cost-of-living index is unity by assumption. The GDP implicit price deflator, on the other hand, is given by:

$$P^P_{t,0} \equiv \frac{\prod_{t,0}}{Y^L_{t,0}} = \frac{p_t y_t - p_{M,t} y_{M,t}}{y_t - y_{M,t}} = \frac{1}{(1+s_{M,t})p_t^{-1} - s_{M,t} p_{M,t}^{-1}}, \tag{11}$$

where s_M is the share of imports in GDP.[10] One finds that the GDP implicit price deflator is equal to $\mathcal{O}\mathcal{B}_1/\mathcal{O}\mathcal{A}_1$, which is greater than unity. That is, the conventional GDP deflator indicates a price increase, even though no disaggregate price has gone up, and one has actually fallen. This bizarre outcome is due to the fact that, as shown by Eq.

[9] The fall in real GDP is due to the fact that the Laspeyres quantity index tends to underestimate the aggregate quantity in the context of production theory, except in the extreme cases of linear or Leontief transformation functions. If one used an index number that is exact for the true production function, real GDP would not change, but it would still fail to pick up the increase in real income. Note also that, in a multi-period framework, a steady improvement in the terms of trade would cause a fall in real GDP even if one used a chained—rather than a direct—Laspeyres quantity index; that is, even if one renormalized prices every period.

[10] The GDP share of gross output therefore is $1+s_M$.

(11), the price of imports enters the calculation of the GDP price deflator with a negative weight.[11]

Just like in the model of the previous section, the difference between the indexes of real GDP and real income is mirrored by the differences in the weights that are being used in the corresponding price deflators. The cost of living is p, the price of the lone output. The price of imports carries no weight since imports are middle products and therefore have no direct impact on the prices faced by final users. The GDP deflator, on the other hand, attributes a weight that is greater than one to p, and a negative weight to p_M. A drop in the price of imports, other things equal, necessarily increases the GDP deflator.[12] Since the GDP deflator overstates the change in the price level that results from an improvement in the terms of trade,[13] it immediately follows that real GDP necessarily underestimate the resulting increase in real income. Another way of looking at the problem is as follows. When import prices fall, the country can afford to import more. Yet, real GDP is obtained by subtracting imports valued at their base period prices. By failing to take into account the lower price of imports, one ends up subtracting too much.

4. Generalization

The model of the previous section is rather restrictive. Fortunately, it can easily be generalized to incorporate technological change and to allow for many inputs and many outputs. In what follows, we assume two outputs—domestic expenditures (or sales) and exports, labeled D and X, respectively—and two primary inputs—labor (L) and capital (K). Domestic factor quantities are denoted by x_j and the corresponding rental prices by w_j ($j \in \{L, K\}$). It is convenient to describe the country's technology by the GDP function that is defined as follows:[14]

$$\pi(p_{D,t}, p_{X,t}, p_{M,t}, x_{L,t}, x_{K,t}, t) \equiv \{\max_{y_D, y_X, y_M} p_{D,t}y_D + p_{X,t}y_X - p_{M,t}y_M : (y_D, y_X, y_M, x_{L,t}, x_{K,t}, t) \in T_t\}, \quad (12)$$

[11] This argument is not as academic as it might seem. A similar situation actually occurred in the United States between the second and the third quarters of 2001 (see the National Income and Products Accounts, Table 1, revision of July 31, 2002). The deflators of all five GDP components fell (consumption, from 109.64 to 109.62; investment, from 100.86 to 100.78; government expenditures, from 113.46 to 113.37; exports, from 96.44 to 95.97; imports, from 94.17 to 89.87), and yet the GDP implicit price index *increased* (from 109.32 to 109.92). This point is also made by Diewert (2002).

[12] P^P increases, even though neither p nor p_M have gone up. The only price that does increase in this example is the nominal (and the real) return to the domestic composite factor. One might be tempted to conclude from this that the GDP price deflator is an index of domestic factor rental prices, rather than of output prices. Unfortunately, this interpretation must be abandoned as soon as one considers a technological progress that, for given output prices, increases domestic factor returns (and real GDP), but, as shown in Section 2, leaves the GDP deflator unchanged.

[13] The negative weight assigned to import prices also implies that the GDP price deflator need not lie within the bounds set by its components. From 1980 to 1996, for instance, the Swiss GDP deflator increased by 65.5%, whereas the price of domestic expenditures went up by 50.2%, the price of exports by 40.1%, and the price of imports by 4.7%.

[14] See Kohli (1978, 1991), and Woodland (1982).

where T_t is the production possibilities set at time t; it is assumed to be a convex cone. The GDP function is linearly homogeneous and convex in prices, and linearly homogeneous and concave in input quantities.

It is well known that the profit-maximizing output supply and import demand functions can be obtained by differentiation:[15]

$$\frac{\partial \pi(\cdot)}{\partial p_i} = \pm y_i(p_{D,t}, p_{X,t}, p_{M,t}, x_{L,t}, x_{K,t}, t), \quad i \in \{D, X, M\}, \tag{13}$$

where the minus sign applies to imports. Moreover, assuming that the domestic factors are mobile between firms, the partial derivatives with respect to the fixed input quantities yield the competitive domestic factor rental prices:

$$\frac{\partial \pi(\cdot)}{\partial x_j} = w_j(p_{D,t}, p_{X,t}, p_{M,t}, x_{L,t}, x_{K,t}, t), \quad j \in \{K, L\}. \tag{14}$$

It is convenient to define g as the inverse terms of trade and h as the relative price of exports, where domestic expenditures are used as the numeraire:

$$g_t \equiv \frac{p_{M,t}}{p_{X,t}} \tag{15}$$

$$h_t \equiv \frac{p_{X,t}}{p_{D,t}}. \tag{16}$$

GDP function (12) can then be rewritten as follows:

$$\pi(\cdot) = \pi(p_{D,t}, h_t p_{D,t}, h_t g_t p_{D,t}, x_{L,t}, x_{K,t}, t) \equiv \psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t). \tag{17}$$

It follows from the properties of $\pi(\cdot)$ that GDP function $\psi(\cdot)$ is linearly homogeneous in p_D. Moreover, one can see from Eqs. (13), (14) and (17) that:

$$\frac{\partial \psi(\cdot)}{\partial p_D} = y_{D,t} + h_t y_{X,t} - h_t g_t y_{M,t} \tag{18}$$

$$\frac{\partial \psi(\cdot)}{\partial h} = p_{D,t}(y_{X,t} - g_t y_{M,t}) \tag{19}$$

$$\frac{\partial \psi(\cdot)}{\partial g} = -p_{X,t} y_{M,t} \tag{20}$$

[15] Again, see Kohli (1978, 1991), and Woodland (1982).

$$\frac{\partial\psi(\cdot)}{\partial x_j} = w_{j,t}, \qquad j\in\{L,K\} \tag{21}$$

$$\frac{\partial\psi(\cdot)}{\partial t} = \frac{\partial\pi(\cdot)}{\partial t}. \tag{22}$$

As shown by Diewert and Morrison (1986), the GDP function is a convenient analytical tool to identify the GDP effect of technological progress. The following index indicates the GDP impact of the passage of time between periods $t-1$ and t, holding all output prices and domestic factor endowments constant:[16]

$$R^L_{t,t-1} \equiv \frac{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)}. \tag{23}$$

Note that in defining Eq. (23) all output prices and domestic input quantities are held constant at their values of period $t-1$. $R^L_{t,t-1}$, thus, has the Laspeyres form, so to speak. Alternatively, one could have frozen output prices and fixed input quantities at their period-t values to obtain the following Paasche-like index of the GDP effect of technological progress:

$$R^P_{t,t-1} \equiv \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t-1)}. \tag{24}$$

Diewert and Morrison (1986) recommend taking the geometric average of the two indexes just defined. This yields the following Fisher-like index of the GDP effect of technological progress:

$$R_{t,t-1} \equiv \sqrt{R^L_{t,t-1} R^P_{t,t-1}}. \tag{25}$$

Similarly for the GDP impact of changes in domestic factor endowments:

$$X_{L,t,t-1} \equiv \sqrt{\frac{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t}, x_{K,t-1}, t-1)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t}, h_t, g_t, x_{L,t-1}, x_{K,t}, t)}}, \tag{26}$$

$$X_{K,t,t-1} \equiv \sqrt{\frac{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t}, t-1)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t-1}, t)}}. \tag{27}$$

[16] All the effects discussed in this and the next section are defined for consecutive periods. Chained indexes valid for longer time intervals can be obtained by compounding.

Next, we can define the following GDP terms-of-trade effect:

$$G_{t,t-1} \equiv \sqrt{\frac{\psi(p_{D,t-1}, h_{t-1}, g_t, x_{L,t-1}, x_{K,t-1}, t-1)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t}, h_t, g_{t-1}, x_{L,t}, x_{K,t}, t)}}, \tag{28}$$

and the GDP trade-balance effect:

$$H_{t,t-1} \equiv \sqrt{\frac{\psi(p_{D,t-1}, h_t, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t}, h_{t-1}, g_t, x_{L,t}, x_{K,t}, t)}}. \tag{29}$$

Some explanations regarding $H_{t,t-1}$, the GDP trade-balance effect, are in order. While an improvement in the terms of trade unambiguously increases real income and welfare, a change in the price of traded goods relative to the price of domestic expenditures can trigger real effects as well. Consider a small equiproportionate increase in the prices of imports and exports thus leaving the terms of trade unchanged. If trade is balanced, the additional export revenues exactly offset the extra cost of the imports. In case of a trade deficit, however, the higher traded-good prices will make the country worse off, while the reverse is true in the case of a surplus. This sort of leverage effect is typically buried in the GDP price deflator, but even though it will generally be small, it is a real effect that deserves to be identified separately.

All five effects just defined are real, and thus they contribute to explaining changes in the country's real domestic income. To square things off, we finally define the GDP domestic-expenditure price effect:

$$P_{D,t,t-1} \equiv \sqrt{\frac{\psi(p_{D,t}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t-1}, h_t, g_t, x_{L,t}, x_{K,t}, t)}}. \tag{30}$$

5. Measurement

Assume that the GDP function has the following translog form:[17]

$$\begin{aligned} \ln\pi_t = {} & \alpha_0 + \sum_i \alpha_i \ln p_{i,t} + \sum_j \beta_j \ln x_{j,t} + \frac{1}{2}\sum_i\sum_h \gamma_{ih}\ln p_{i,t}\ln p_{h,t} \\ & + \frac{1}{2}\sum_j\sum_k \phi_{jk}\ln x_{j,t}\ln x_{k,t} + \sum_i\sum_j \delta_{ij}\ln p_{i,t}\ln x_{j,t} + \sum_i \delta_{iT}\ln p_{i,t}t \\ & + \sum_j \phi_{jT}\ln x_{j,t}t + \beta_T t + \frac{1}{2}\phi_{TT}t^2, \;\; i,h \in \{D,X,M\};\; j,k \in \{L,K\}, \end{aligned} \tag{31}$$

where $\Sigma\alpha_i = 1$, $\Sigma\beta_j = 1$, $\gamma_{ih} = \gamma_{hi}$, $\phi_{jk} = \phi_{kj}$, $\Sigma\gamma_{ih} = 0$, $\Sigma\phi_{jk} = 0$, $\Sigma_i\delta_{ij} = 0$, $\Sigma_j\delta_{ij} = 0$, $\Sigma\delta_{iT} = 0$, and $\Sigma\phi_{jT} = 0$.

[17] The translog function gives a second-order approximation in logarithms to an arbitrary GDP function; see Christensen et al. (1973), and Diewert (1974).

We show in Appendix A that if GDP function $\pi(\cdot)$ is translog, then GDP function $\psi(\cdot)$ defined by Eq. (17) is translog as well. That is, $\psi(\cdot)$ provides a flexible representation of the country's technology.

If estimates of the translog GDP function were available, it would be a simple matter to calculate the various effects defined in the previous section.[18] However, it turns out that as long as the true GDP function is translog, all these effects can be calculated from the data alone; that is, without needing to know the values of the parameters of the GDP function. Thus, we show in Appendix A that:

$$R_{t,t-1} = \frac{\prod_{t,t-1}}{P_{t,t-1} \cdot X_{t,t-1}}, \tag{32}$$

where $\Pi_{t,t-1}$ is once again the growth factor of nominal GDP, $P_{t,t-1}$ is a Törnqvist price index of the GDP output components (including imports), and $X_{t,t-1}$ is a Törnqvist quantity index of domestic factor endowments:

$$\begin{aligned}\prod_{t,t-1} &\equiv \frac{\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t)}{\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t-1)} \\ &= \frac{p_{D,t}y_{D,t} + p_{X,t}y_{X,t} - p_{M,t}y_{M,t}}{p_{D,t-1}y_{D,t-1} + p_{X,t-1}y_{X,t-1} - p_{M,t-1}y_{M,t-1}}\end{aligned} \tag{33}$$

$$P_{t,t-1} \equiv \exp\left[\sum_i \pm \frac{1}{2}(s_{i,t} + s_{i,t-1})\ln\frac{p_{i,t}}{p_{i,t-1}}\right], \quad i \in \{D, X, M\} \tag{34}$$

$$X_{t,t-1} \equiv \exp\left[\sum_j \frac{1}{2}(s_{j,t} + s_{j,t-1})\ln\frac{x_{j,t}}{x_{j,t-1}}\right], \quad j \in \{L, K\}, \tag{35}$$

where the sign in Eq. (34) is negative for imports and positive otherwise. Similarly, one can show that:[19]

$$X_{j,t,t-1} = \exp\left[\frac{1}{2}(s_{j,t} + s_{j,t-1})\ln\frac{x_{j,t}}{x_{j,t-1}}\right], \quad j \in \{L, K\} \tag{36}$$

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[18] See Kohli (1990, 1991) for such an econometric approach.

[19] See Appendix A. Our measure of the terms-of-trade effect—see (37) below—is different from the one proposed by Diewert and Morrison (1986), and which we have used in previous work; see Kohli (1990, 1991), for instance:

$$A_{t,t-1} \equiv \exp\left[\sum_i \pm \frac{1}{2}(s_{i,t} + s_{i,t-1})\ln\frac{p_{i,t}}{p_{i,t-1}}\right], \quad i \in \{X, M\}.$$

This measure raises some difficulties, however. Thus, if the prices of imports and exports increase in the same proportions (following a devaluation of the national currency, for instance), $A_{t,t-1}$ registers a change unless trade happens to be balanced on average over the two periods, even though the terms of trade clearly do not change in such a case. Put differently, $A_{t,t-1}$, which is meant to measure a real effect, is generally not homogeneous of degree zero in prices. This implies that the element that is supposed to measure the contribution of prices in the GDP growth decomposition will generally not be linearly homogeneous in prices.

$$G_{t,t-1} = \exp\left[\frac{1}{2}(-s_{M,t} - s_{M,t-1})\ln\frac{g_t}{g_{t-1}}\right] \tag{37}$$

$$H_{t,t-1} = \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1})\ln\frac{h_t}{h_{t-1}}\right] \tag{38}$$

$$P_{D,t,t-1} = \frac{p_{D,t}}{p_{D,t-1}}, \tag{39}$$

where $s_B \equiv s_X - s_M$.

Finally, we show in Appendix A that the six effects that we just obtained together give a *complete* decomposition of the growth in nominal GDP:

$$\prod\nolimits_{t,t-1} = P_{D,t,t-1} \cdot H_{t,t-1} \cdot G_{t,t-1} \cdot X_{L,t,t-1} \cdot X_{K,t,t-1} \cdot R_{t,t-1}. \tag{40}$$

It is noteworthy that the product of the first three terms on the right-hand side of Eq. (40) yields the Törnqvist price index defined by Eq. (34):

$$P_{t,t-1} = P_{D,t,t-1} \cdot H_{t,t-1} \cdot G_{t,t-1}. \tag{41}$$

In other words, the remaining three terms together make up a measure of what is an implicit Törnqvist index of real GDP ($\tilde{Y}_{t,t-1}$):[20]

$$\tilde{Y}_{t,t-1} \equiv \frac{\prod_{t,t-1}}{P_{t,t-1}} = X_{L,t,t-1} \cdot X_{K,t,t-1} \cdot R_{t,t-1}. \tag{42}$$

Such an implicit Törnqvist index of real GDP would be much preferable to the Laspeyres index commonly used, in that it is a superlative index.[21] Nevertheless, it excludes the terms-of-trade and the trade-balance effects that we have defined earlier and that we have repeatedly characterized as real—rather than price—effects. These considerations lead us to define the following index of real domestic income ($\tilde{Q}_{t,t-1}$) obtained by combining all five real effects contained in Eq. (40):

$$\tilde{Q}_{t,t-1} \equiv H_{t,t-1} \cdot G_{t,t-1} \cdot X_{L,t,t-1} \cdot X_{K,t,t-1} \cdot R_{t,t-1} = \tilde{Y}_{t,t-1} \cdot H_{t,t-1} \cdot G_{t,t-1} = \frac{\prod_{t,t-1}}{P_{D,t,t-1}}. \tag{43}$$

It is rather remarkable that $\tilde{Q}_{t,t-1}$, which captures the combined effect of five real forces, can be measured simply by deflating the change in nominal GDP by the index measuring the change in the price of domestic expenditures; that is, without needing any price and quantity data for labor and capital.

[20] See Kohli (1999).

[21] The implicit Törnqvist index of real GDP is numerically very close to the Fisher index recently adopted by the United States and a few other countries. Note, however, that the Fisher index is not exact for any known GDP function, except under rather restrictive conditions such as global separability between outputs (including imports) and domestic inputs.

6. Command-Basis GDP

Since 1981, the U.S. Bureau of Economic Analysis publishes series of what has become known as "Command-Basis" GNP. Command-Basis GNP (GDP) is a measure of real GNP (GDP) that tries to take into account the effects of changes in the terms of trade on the purchasing power of a nation.[22] Thus, instead of deflating nominal imports by the price of imports and nominal exports by the price of exports, the entire trade balance (i.e. net exports) is deflated by the same price index. Choosing the import price deflator for this purpose amounts to replacing constant dollar exports by the import equivalent of exports when adding up the various components that make up real GDP. The idea behind this procedure is that what matters is not the quantity of goods and services that is being exported, but rather the quantity of imports that are made possible through these exports. Formally, Command-Basis GDP ($B_{t,0}^L$) can be calculated as:[23]

$$B_{t,0}^L \equiv \frac{y_{D,t} + y_{X,t}(p_{X,t}/p_{M,t}) - y_{M,t}}{y_{D,0} + y_{X,0} - y_{M,0}}. \tag{44}$$

One obvious question that arises when it comes to Command-Basis GDP concerns the choice of the price index used to deflate the trade account. Why use the import price deflator? Why not use the export price deflator? Or an average of the two? Or the GDP deflator? To the extent that the trade balance is close to zero, or if the terms of trade remain little changed, this choice does not matter much. Nevertheless, our approach suggests an answer to this question, an answer that rests on a solid theoretical foundation. Thus, expression (43) indicates that the same price index should be used to deflate the trade account and the value of domestic outputs, that it should be based on the domestic price components alone, and that a superlative index formula should be preferred.[24]

7. Trading gains and losses

As mentioned earlier, official statisticians are well aware of the distinction between real GDP and real domestic income as a consequence of changing terms of trade. Using p_D as a deflator, a simple estimate of the difference between the two concepts is given by the following standard measure of the trading gains or losses (γ_t^L):[25]

$$\gamma_t^L \equiv \frac{\pi_t}{p_{D,t}} - (y_{D,t} + y_{X,t} - y_{M,t}) = y_{X,t}\left(\frac{p_{X,t}}{p_{D,t}} - 1\right) - y_{M,t}\left(\frac{p_{M,t}}{p_{D,t}} - 1\right). \tag{45}$$

[22] Although the concept was originally introduced in the context of GNP, today it is often used for GDP as well; see Dewald (1995), for instance.

[23] See Denison (1981).

[24] Alternatively, one could use a Fisher price index. This would be more in line with current U.S. practices. Although the Fisher index is not be exact for the translog GDP function, it would yield results that numerically would be very close.

[25] See United Nations (2002), paragraphs 16.151–152; a positive value of γ_t^L denotes a gain and a negative one a loss.

One difficulty with this measure is that it depends on the normalization of the data, i.e. on the choice of the base period. A second difficulty is that, being expressed in absolute terms, it is difficult to interpret its size. On these grounds, one might prefer the following trading-gains index, defined with reference to a well-defined base period (period 0) and relative to real GDP:

$$\Gamma^L_{t,0} \equiv \frac{\prod_{t,0} \Big/ p_{D,t,0}}{Y^L_{t,0}} = \frac{y_{D,t} + y_{X,t}p_{X,t}/p_{D,t} - y_{M,t}p_{M,t}/p_{D,t}}{y_{D,t} + y_{X,t} - y_{M,t}}. \tag{46}$$

Both Eqs. (45) and (46) are defined with reference to the direct Laspeyres index of real GDP ($Y^L_{t,0}$). A superlative measure of the trading gains (or losses) can be obtained by using instead the chained implicit Törnqvist index of real GDP defined by Eq. (42). We thus get the following Törnqvist-based trading-gains index:

$$\Gamma_{t,t-1} \equiv \frac{\tilde{Q}_{t,t-1}}{\tilde{Y}_{t,t-1}} = G_{t,t-1} \cdot H_{t,t-1}, \tag{47}$$

where we have made use of Eq. (43). We find that the trading gains consist of two parts, the terms-of-trade effect and the trade-balance effect. Moreover, these two effects, as defined by Eqs. (28) and (29), give a *complete* decomposition of the Törnqvist trading-gains index. Making use of Eqs. (41) and (47), finally, one gets:

$$\Gamma_{t,t-1} = \frac{P_{t,t-1}}{P_{D,t,t-1}}. \tag{48}$$

That is, the Törnqvist trading-gains index can be obtained directly as the ratio of the Törnqvist GDP price index to the domestic-expenditure price index.

8. International comparisons

How important is the distinction between real GDP (conventional measure) and the implicit Törnqvist measure of real domestic income given by Eq. (43)? The answer to this empirical question will depend to a large extent on the terms-of-trade changes (and especially the improvements) that a country experiences over time, and on the size of its foreign sector. We report in Table 1 growth estimates for a number of industrialized nations.[26] The first column of the table reports cumulated growth for the period 1980–1996 based on the direct Laspeyres index of real GDP (Eq. (10)). The second column does the same using the index of Command-Basis GDP (Eq. (44)) instead. The estimates in the third column are based on the implicit Törnqvist index of real GDP (47), whereas the estimates in the fourth column are based on our implicit Törnqvist index of real domestic income (43).[27]

[26] All data are drawn from the OECD (Organisation for Economic Cooperation and Development) National Accounts, Main Aggregates, except the ones for the United States which come directly from the Bureau of Economic Analysis. The price of domestic expenditures (p_D) is computed as Törnqvist index of the prices of consumption, investment and government expenditures.

[27] The estimates for Germany are obtained by splicing the West-German growth rates for the period 1980–1991 with those for the entire country for the remaining years.

Table 1
Cumulated growth, international comparisons, 1980–1996

	Laspeyres real GDP (%)	Command-basis GDP (%)	Implicit Törnqvist real GDP (%)	Implicit Törnqvist real income (%)
United States	60.1	61.0	59.4	61.5
Canada	45.4	46.9	45.0	46.0
Mexico	35.7	25.5	35.5	25.6
Japan	65.2	68.7	68.2	68.9
South Korea	265.5	263.8	272.2	277.8
Australia	61.0	61.8	61.1	62.0
New Zealand	40.6	47.3	41.5	49.4
Austria	40.8	39.9	40.9	39.8
Belgium	29.7	34.1	29.4	32.8
Denmark	38.4	38.5	39.0	38.1
Finland	36.6	40.9	37.0	42.4
France	32.5	36.6	32.0	36.2
Germany	39.8	46.6	40.5	43.9
Greece	30.2	32.5	27.5	36.8
Iceland	42.5	39.2	41.6	39.0
Ireland	106.1	99.2	111.1	105.0
Italy	32.8	37.1	31.9	37.9
Luxembourg	107.7	98.3	107.6	96.5
Netherlands	42.2	42.7	43.0	41.6
Norway	59.5	42.7	59.2	39.7
Portugal	47.3	52.4	48.7	60.0
Spain	46.5	49.9	45.7	50.8
Sweden	26.5	26.5	26.4	26.6
Switzerland	22.0	35.0	22.1	34.5
Turkey	108.3	105.6	117.4	111.3
United Kingdom	41.6	41.8	40.9	41.3

Comparing first the figures for the direct Laspeyres indexes of real GDP (column 1) with those of the chained implicit Törnqvist indexes (column 3), we find that the differences are generally quite small, except for countries having experienced high growth, such as South Korea, Ireland, and Turkey. Nonetheless, one would expect the implicit Törnqvist indexes to be more accurate, given that they are superlative indexes in the sense of Diewert (1976), and thus they take full account of the transformation possibilities inherent to the aggregate technology. One would also expect these indexes to be numerically very close to the Fisher indexes of real GDP that have recently been adopted by a number of countries, including the United States.

Next, we can compare the estimates of the implicit Törnqvist indexes of real GDP (column 3) with those of real domestic income (column 4). The differences there are directly imputable to the changes in the prices of traded goods, captured by the terms-of-trade effect (37) and the trade-balance effect (38). We find that real GDP severely underestimates growth in real domestic income in the case of New Zealand, Greece, Italy, Portugal, and Switzerland, countries that have enjoyed significant improvements in their terms of trade over the past two decades. As far as Switzerland is concerned, real GDP underestimates the growth in real domestic income by about 0.6% annually. Over our sample period, this adds up to more than 10% of GDP. Real GDP, on the other hand, significantly overestimates real-income growth in the case of Mexico, Luxembourg, and Norway since it does not take into account the deterioration in their terms of trade. The differences are much smaller for the other countries that have not experienced a secular movement in their terms of trade, or whose foreign sectors are relatively small.

Comparing next the estimates in columns 2 and 4, we find that Command-Basis GDP underestimates real domestic income in countries having experienced high growth, such as South Korea, Ireland, and Turkey. For most of the other countries, the growth rates based on the implicit Törnqvist index of real domestic income are relatively close to the ones obtained with the index of Command-Basis GDP, in spite of the shortcomings of the latter measure as noted above. This is because Command-Basis GDP does attempt to incorporate the effects of terms-of-trade changes, even if it does so in a rather crude and ad hoc manner.

As shown by Eq. (47), the discrepancy between the growth estimates reported in columns 3 and 4 is solely imputable to the trading-gains effect, itself made-up by the terms-of-trade and the trade-balance effects. We show in Table 2 estimates of the cumulated trading gains over the period 1980–1996, based on Eq. (47) and expressed in percentages. We also show

Table 2
Törnqvist trading gains, international comparisons, 1980–1996

	Cumulated trading gain (%)	Minimum yearly value (%)	Maximum yearly value (%)
United States	1.30	– 0.31	0.43
Canada	0.69	– 0.81	0.86
Mexico	– 7.29	– 4.00	1.68
Japan	0.46	– 0.62	1.75
South Korea	1.49	– 2.52	2.03
Australia	0.59	– 1.69	2.28
New Zealand	5.61	– 1.27	2.21
Austria	– 0.84	– 1.45	0.54
Belgium	2.61	– 2.38	2.68
Denmark	– 0.64	– 1.51	1.45
Finland	3.97	– 0.83	1.61
France	3.20	– 0.60	2.19
Germany	2.42	– 1.50	2.86
Greece	7.32	– 1.08	2.20
Iceland	– 1.81	– 1.63	1.95
Ireland	– 2.86	– 3.18	2.08
Italy	4.54	– 0.84	2.64
Luxembourg	– 5.36	– 2.88	3.21
Netherlands	– 1.02	– 1.18	1.08
Norway	– 12.26	– 7.54	1.88
Portugal	7.66	– 2.03	4.33
Spain	3.50	– 1.81	2.50
Sweden	0.14	– 1.09	1.74
Switzerland	10.14	– 0.89	3.77
Turkey	– 2.80	– 1.45	1.68
United Kingdom	0.29	– 1.11	0.45

for each country the minimum and maximum annual values of the trading gains. This makes it possible to better assess the difference between the growth picture based on real GDP and the one that relies on real income. Even though, as noted earlier, the cumulated estimates for the entire sample period are quite close to one another for most countries, this hides large yearly discrepancies. Thus, real GDP has underestimated the annual growth in real domestic income by as much as 4.3% in Portugal, 3.8% in Switzerland, and 3.2% in Luxembourg. It has overestimated annual real-income growth by as much as 7.5% in Norway, 4.0% in Mexico, and 3.2% in Ireland. In the case of Luxembourg, the gap between the two growth rates has been very wide, extending from − 2.9% to 3.2%. These are not trivial magnitudes, and they can severely distort one's assessment of a country's short-term economic performance. Even for the remaining countries we find that the discrepancies are not insignificant. In fact, the difference has been greater than 1% in absolute value at least once in 24 out of the 26 countries in our sample.

9. Concluding comments

The GDP deflator is often described as the broadest measure of a country's price level. When the national-accounts data are published, changes in the GDP deflator are closely scrutinized. Increases are generally viewed with concern since they are interpreted as revealing inflationary pressures. This is clearly inappropriate if the increase in the GDP deflator results from a *decrease* in import prices. A fall in import prices, other things equal, is fundamentally a positive outcome. It has no inflationary effects, quite the contrary. If an increase in the GDP deflator is incorrectly interpreted as signaling inflationary pressures, it could lead to the wrong policy reaction. Yet, it is precisely the GDP deflator that Taylor (1993) used as a measure of inflation when proposing his famous rule for monetary policy.[28]

For many countries, including the United States, the difference between real GDP and the implicit Törnqvist index of real domestic income appears to be rather small on average. Nevertheless, a difference of one or two percentage points of real growth is not trivial: for the United States, using 1980 as a starting point, the difference by 1996 amounted to about 1.3% of GDP, i.e. over 100 billion dollars at 1996 prices. Such figures are of the same order of magnitude as the gap between real GDP and Command-Basis GDP. This discrepancy was deemed large enough by the U.S. Bureau of Economic Analysis to warrant publication of Command-Basis GNP series. Moreover, long-run averages can mask significant annual deviations. Thus, the annual growth discrepancy has exceeded one percentage point at least once for most countries in our sample. Since annual changes in economic performance are closely watched and may trigger policy responses, it is important that they be measured as accurately as possible. This is particularly easy to do here since the data necessary to compute the implicit Törnqvist real-income index are exactly the same as the ones needed to construct real GDP.

[28] The use of the GDP deflator as an inflation target is rejected by Diewert (2002) also.

Real GDP was found to underestimate the growth in real domestic income in a majority of the countries in our sample. This is due to the improvements in the terms of trade that these countries have experienced over the past two decades. Since our sample was made up mostly by industrialized nations, it suggests that many less-developed countries must have suffered a worsening in their own terms of trade. Consequently, real GDP will tend to overstate the increase in real income in those countries. Any evidence on income convergence between poor and rich countries, which typically rests on real GDP comparisons, may thus have to be reassessed.

One could object that even if it is true that real income increases as a result of an improvement in the terms of trade, this is of limited interest since it might not create a single job. The reason why many economists are interested in GDP figures is because an increase in real GDP is usually associated with a rise in employment. Even if it were true that an improvement in the terms of trade does not create any jobs,[29] this criticism would be beside the point for several reasons. For a start, as we showed above, an improvement in the terms of trade may lead to a reduction in real GDP, a reduction that is meaningless. Second, a technological change, which is integrated in the calculation of real GDP, leads to an increase in real GDP without necessarily creating any jobs either. Technological progress, just like an improvement in the terms of trade, may be pro- or anti-labor biased. Both phenomena are similar, and there is no reason therefore to treat them differently. In truth, if one was really interested in the demand for labor, it would be much more sensible to derive it from a GDP function such as Eq. (12), instead of relying on a very crude indicator such as real GDP.[30] Finally, one ought to remember that it is real income—and ultimately consumption—that generates utility, rather than work effort.

One could also counter that real GDP attempts to measure a country's production effort—production requires hard work—and that there is little merit in experiencing an "effortless" improvement in the terms of trade. Such an objection would reveal an exceedingly narrow view of the nature of production activities. While an improvement in the terms of trade may indeed be a purely exogenous event, foreign trade is an activity that requires much effort. Importers and exporters must constantly scout world markets in search of better opportunities, domestic producers must position themselves to exploit existing comparative advantages, and always be on the lookout for new ones. These efforts require resources, and they are an intimate part of the production process. Similar considerations apply to technological change. Technological progress too may be the outcome of chance, or it may even be imported from abroad. It is certainly not true that every invention or innovation is the outcome of a systematic and tiresome research effort. There is therefore no reason to discriminate between these two types of efforts on a a priori basis. This is all the more true that, as we argued earlier, it may be impossible to distinguish one from the other.

[29] Note that in the context of the GDP-function model, employment is exogenous; it is the return to labor that is endogenous. Models that treat employment as endogenous can be found in Kohli (1991) who finds that, for the United States, a drop in the price of imports increases the demand for labor.

[30] The inverse demand for labor Eq. (14) derived from the GDP function model is much more sophisticated than most specifications commonly used in the literature, and it shows that both changes in the terms of trade and technological progress are likely to affect wages.

Acknowledgements

I am grateful to W. Erwin Diewert, Andreas M. Fischer, Kevin J. Fox, Jean-Christian Lambelet, Peter Stalder, and two anonymous referees for helpful comments and suggestions. Needless to say, they are not responsible for any errors or omissions.

Appendix A

We begin by showing that if $\pi(\cdot)$ is translog, then $\psi(\cdot)$ is translog too. GDP function (31) can be rewritten as follows (the time subscript is omitted for clarity):

$$\begin{aligned}
\ln\pi = {} & \alpha_0 + \alpha_D \ln p_D + \alpha_X \ln p_X + \alpha_M \ln p_M + \frac{1}{2}\gamma_{DD}(\ln p_D)^2 + \frac{1}{2}\gamma_{XX}(\ln p_X)^2 \\
& + \frac{1}{2}\gamma_{MM}(\ln p_M)^2 + \gamma_{DX}\ln p_D \ln p_X + \gamma_{DM}\ln p_D \ln p_M + \gamma_{XM}\ln p_X \ln p_M \\
& + \sum \delta_{Dj}\ln p_D \ln x_j + \sum \delta_{Xj}\ln p_X \ln x_j + \sum \delta_{Mj}\ln p_M \ln x_j + \delta_{DT}\ln p_D t \\
& + \delta_{XT}\ln p_X t + \delta_{MT}\ln p_M t + \sum \beta_j \ln x_j + \frac{1}{2}\sum\sum \phi_{jk}\ln x_j \ln x_k + \sum \phi_{jT}\ln x_j t \\
& + \beta_T t + \frac{1}{2}\phi_{TT}t^2 = \alpha_0 + (\alpha_D + \alpha_X + \alpha_M)\ln p_D + (\alpha_X + \alpha_M)(\ln p_X - \ln p_D) \\
& + \alpha_M(\ln p_M - \ln p_X) + \frac{1}{2}(\gamma_{DD} + \gamma_{XX} + \gamma_{MM} + 2\gamma_{DX} + 2\gamma_{DM} + 2\gamma_{XM})(\ln p_D)^2 \\
& + \frac{1}{2}(\gamma_{XX} + \gamma_{MM} + 2\gamma_{XM})(\ln p_X - \ln p_D)^2 + \frac{1}{2}\gamma_{MM}(\ln p_M - \ln p_X)^2 \\
& + (\gamma_{XX} + \gamma_{MM} + \gamma_{DX} + \gamma_{DM} + 2\gamma_{XM})\ln p_D(\ln p_X - \ln p_D) \\
& + (\gamma_{MM} + \gamma_{DM} + \gamma_{XM})\ln p_D(\ln p_M - \ln p_X) + (\gamma_{MM} + \gamma_{XM})(\ln p_X - \ln p_D) \\
& \times (\ln p_M - \ln p_X) + [(\delta_{DL} + \delta_{XL} + \delta_{ML})\ln x_L + (\delta_{DK} + \delta_{XK} + \delta_{MK})\ln x_K]\ln p_D \\
& + [(\delta_{XL} + \delta_{ML})\ln x_L + (\delta_{XK} + \delta_{MK})\ln x_K](\ln p_X - \ln p_D) \\
& + (\delta_{ML}\ln x_L + \delta_{MK}\ln x_K)(\ln p_M - \ln p_X) + (\delta_{DT} + \delta_{XT} + \delta_{MT})\ln p_D t \\
& + (\delta_{XT} + \delta_{MT})(\ln p_X - \ln p_D)t + \delta_{MT}(\ln p_M - \ln p_X)t + \sum_j \beta_j \ln x_j \\
& + \frac{1}{2}\sum\sum \phi_{jk}\ln x_j \ln x_k + \sum_j \phi_{jT}\ln x_j t + \beta_T t + \frac{1}{2}\phi_{TT}t^2 \\
= {} & \alpha_0 + a_D \ln p_D + a_H \ln h + a_G \ln g + \frac{1}{2}c_{DD}(\ln p_D)^2 + \frac{1}{2}c_{HH}(\ln h)^2 + \frac{1}{2}c_{GG}(\ln g)^2 \\
& + c_{DH}\ln p_D \ln h + c_{DG}\ln p_D \ln g + c_{HG}\ln h \ln g + \sum d_{Dj}\ln p_D \ln x_j + \sum d_{Hj}\ln h \ln x_j \\
& + \sum d_{Gj}\ln g \ln x_j + d_{DT}\ln p_D t + d_{HT}\ln h t + d_{GT}\ln g t + \sum \beta_j \ln x_j \\
& + \frac{1}{2}\sum\sum \phi_{jk}\ln x_j \ln x_k + \sum \phi_{jT}\ln x_j t + \beta_T t + \frac{1}{2}\phi_{TT}t^2, \qquad (49)
\end{aligned}$$

where $a_D = \alpha_D + \alpha_X + \alpha_M = 1$, $a_H = \alpha_X + \alpha_M$, $a_G = \alpha_M$, $c_{DD} = \gamma_{DD} + \gamma_{XX} + \gamma_{MM} + 2\gamma_{DX} + 2\gamma_{DM} + 2\gamma_{XM} = 0$, $c_{HH} = \gamma_{XX} + \gamma_{MM} + 2\gamma_{XM}$, $c_{GG} = \gamma_{MM}$, $c_{DH} = \gamma_{XX} + \gamma_{MM} + \gamma_{DX} + \gamma_{DM} + 2\gamma_{XM} = 0$, $c_{DG} = \gamma_{MM} + \gamma_{DM} + \gamma_{XM} = 0$, $c_{HG} = \gamma_{MM} + \gamma_{XM}$, $d_{Dj} = \delta_{Dj} + \delta_{Xj} + \delta_{Mj} = 0$, $d_{Hj} = \delta_{Xj} + \delta_{Mj}$, $d_{Gj} = \delta_{Mj}$, $j \in \{L, K, T\}$. This shows that $\psi(\cdot)$ is a translog function in p_D, h, g, x_L, x_K and t. Moreover, as expected, it is linearly homogeneous in p_D.

The logarithmic derivatives of $\psi(\cdot)$ with respect to its arguments are as follows:

$$\frac{\partial \ln\psi(\cdot)}{\partial \ln p_D} = a_D + c_{DD}\ln p_D + c_{DH}\ln h + c_{DG}\ln g + d_{DL}\ln x_L + d_{DK}\ln x_K + d_{DT}t = 1 \tag{50}$$

$$\frac{\partial \ln\psi(\cdot)}{\partial \ln h} = a_H + c_{DH}\ln p_D + c_{HH}\ln h + c_{HG}\ln g + d_{HL}\ln x_L + d_{HK}\ln x_K + d_{HT}t = s_B \tag{51}$$

$$\frac{\partial \ln\psi(\cdot)}{\partial \ln g} = a_G + c_{DG}\ln p_D + c_{HG}\ln h + c_{GG}\ln g + d_{GL}\ln x_L + d_{GK}\ln x_K + d_{GT}t = -s_M \tag{52}$$

$$\begin{aligned}\frac{\partial \ln\psi(\cdot)}{\partial \ln x_j} &= \beta_j + d_{Dj}\ln p_D + d_{Hj}\ln h + d_{Gj}\ln g + \phi_{jL}\ln x_L + \phi_{jK}\ln x_K + \phi_{jT}t \\ &= s_j \quad j \in \{L, K\}\end{aligned} \tag{53}$$

$$\frac{\partial \ln\psi(\cdot)}{\partial t} = \beta_T + d_{DT}\ln p_D + d_{HT}\ln h + d_{GT}\ln g + \phi_{LT}\ln x_L + \phi_{KT}\ln x_K + \phi_{TT}t = s_T \tag{54}$$

Next, it is useful to consider the change in the value of the GDP function between consecutive periods:

$$\begin{aligned}
&\ln \prod\nolimits_{t,t-1} \\
&= \ln\psi(z_t) - \ln\psi(z_{t-1}) = \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln p_D} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln p_D}\right](\ln p_{D,t} - \ln p_{D,t-1}) \\
&\quad + \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln h} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln h}\right](\ln h_t - \ln h_{t-1}) + \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln g} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln g}\right] \\
&\quad \times (\ln_t - \ln g_{t-1}) + \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln x_L} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln x_L}\right](\ln x_{L,t} - \ln x_{L,t-1}) \\
&\quad + \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln x_K} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln x_K}\right](\ln x_{K,t} - \ln x_{K,t-1}) + \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial t} + \frac{\partial \ln\psi(z_{t-1})}{\partial t}\right] \\
&= (\ln p_{D,t} - \ln p_{D,t-1}) + \frac{1}{2}(s_{B,t} + s_{B,t-1})(\ln h_t - \ln h_{t-1}) + \frac{1}{2}(-s_{M,t} - s_{M,t-1}) \\
&\quad \times (\ln g_t - \ln g_{t-1}) + \frac{1}{2}(s_{L,t} + s_{L,t-1})(\ln x_{L,t} - \ln x_{L,t-1}) + \frac{1}{2}(s_{K,t} + s_{K,t-1}) \\
&\quad \times (\ln x_{K,t} - \ln x_{K,t-1}) + \frac{1}{2}(s_{T,t} + s_{T,t-1}),
\end{aligned} \tag{55}$$

where $z_t \equiv [p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t]'$, and where we have made use of the Quadratic Approximation Lemma—see Diewert (1976)—and of Eqs (50)–(54).

Next, we can provide the details of the derivation of Eq. (37). Starting with expression (28) we get:

$$\begin{aligned}\ln G_{t,t-1} &= \frac{1}{2}[\ln\psi(p_{D,t-1}, h_{t-1}, g_t, x_{L,t-1}, x_{K,t-1}, t-1) + \ln\psi(z_t) - \ln\psi(z_{t-1}) \\ &\quad -\ln\psi(p_{D,t}, h_t, g_{t-1}, x_{L,t}, x_{K,t}, t)] \\ &= \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial \ln g} + \frac{\partial \ln\psi(z_{t-1})}{\partial \ln g}\right](\ln g_t - \ln g_{t-1}) \\ &= \frac{1}{2}(-s_{M,t} - s_{M,t-1})(\ln g_t - \ln g_{t-1}), \end{aligned} \tag{56}$$

where we have applied the translog quadratic identity—see Caves et al. (1982)—as well as Eq. (52). This demonstrates the validity of (37). The proofs for Expressions (36), (38), and (39) proceeds along exactly the same lines, but they are omitted here for lack of space. For technological change, finally, we get:

$$\begin{aligned}\ln R_{t,t-1} &= \frac{1}{2}[\ln\psi(p_{D,t-1}, h_{t-1}, g_{t-1}, x_{L,t-1}, x_{K,t-1}, t) + \ln\psi(z_t) - \ln\psi(z_{t-1}) \\ &\quad - \ln\psi(p_{D,t}, h_t, g_t, x_{L,t}, x_{K,t}, t-1)] \\ &= \frac{1}{2}\left[\frac{\partial \ln\psi(z_t)}{\partial t} + \frac{\partial \ln\psi(z_{t-1})}{\partial t}\right] \\ &= \frac{1}{2}(s_{T,t} + s_{T,t-1}) \\ &= \ln \prod\nolimits_{t,t-1} - \ln P_{D,t,t-1} - \ln H_{t,t-1} - \ln G_{t,t-1} - \ln X_{L,t,t-1} - \ln X_{K,t,t-1} \\ &= \ln \prod\nolimits_{t,t-1} - \ln P_{t,t-1} - \ln X_{t,t-1}, \end{aligned} \tag{57}$$

where we have made use of Eqs. (54) and (55) and Eqs. (34)–(39). Note that the second-last line of Eq. (57) also provides the proof of Eq. (40).

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