HOW USEFUL IS THE NEUTRAL INTEREST RATE FOR MONETARY POLICY IN CANADA?[1]

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ABSTRACT

Recently, there has been a renewed interest in using the neutral interest rate for monetary policy. While the neutral rate is theoretically sound, how useful the neutral rate is for the conduct of monetary policy will depend on how well we are measuring it since it is unobservable, and the types of questions we are asking. In this paper, we present some estimates of the neutral rate for Canada based on several different methodologies commonly found in the literature. We then calculate the real interest rate gap, defined as the difference between the actual real interest rate and the derived neutral interest rate. In assessing the usefulness of the neutral rate for monetary policy, we examine their leading information properties for future economic activity and how well they explain past policy actions. We also analyze the effect of uncertainties and data revisions on the neutral rate estimates.

JEL Classification:

PRELIMINARY AND INCOMPLETE

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"[t]here is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest on capital." (Wicksell, 1936, p.102)

"Conceptually, I think there probably is (a neutral interest rate), but I think as a practical matter, using it as a kind of guide to policy, I am very doubtful of that. We have done that often in the past and we have been totally misled" (Sir Edward George, Governor of the Bank of England testimony to the Select Committee on Treasury, June 2002)

"No sir, we have not revised it (the neutral interest rate) up because we do not know what the neutral interest rate is. We do not even, not even internally, use an agreed estimate of what the neutral interest rate is supposed to be." (Wim Duisenberg, Governor of the ECB, Hearing before the Committee on Economic and Monetary Affairs of the European Parliament, June 2000)

1. Introduction

In recent years, the notion of equilibrium or neutral interest rates has regained prominence, both in policy circles (e.g. Blinder, 1998, Julius, 1998, Meyer, 2000) and academia (e.g. Woodford, 2000, Bomfim, 2001, Laubach and Williams, 2001, and Neiss and Nelson, 2003). For example, in 2004, several members of the US Federal Reserve Board, including Alan Greenspan, have made reference to the term in public speeches and testimonies. While policymakers generally acknowledge that it is difficult to know the level of the neutral rate with precision, they tend to find it useful in the communication with the public when the current level of the policy rate clearly deviates from its neutral level, thus suggesting where they see interest rates should be going.

We believe that the neutral interest rate is a relevant concept for monetary policy in Canada, providing a broad indication of the level of real interest rates where monetary policy is neither contractionary nor expansionary. In an era where the primary policy instrument in most countries is the level of short-term interest rates, comparing the level of such a rate relative to some equilibrium, or neutral rate is a useful method to measure the stance of monetary policy.[4] The gap between the current level of interest rate and the equilibrium real rate, usually known as the interest rate gap can thus serve as a barometer to gauge whether policy is stimulative or not.For this concept to be useful, however, policymakers must have a reliable measure of the neutral rate and must be able to understand the driving forces behind the fluctuations in it. Nevertheless, it is important to remember that the real interest rate and its accompanied interest rate gap should be one of the many sources of information that a central bank examines when formulating monetary policy.

So far, the concept of the natural interest rate and the interest rate gap has not been introduced in actual monetary policy formulation in any systematic way.[5] One of the main reasons for this is that the neutral interest rate is not directly observable and thus has to be estimated, usually with a lot of uncertainty. Moreover, there is no agreed-upon definition of the neutral rate and the estimates depend a lot on the estimation methods.

How useful the neutral rate is for the conduct of monetary policy will depend on how well we are measuring it and the types of questions we are asking. In this paper, we present some estimates of the neutral rate for Canada based on several different methodologies commonly found in the literature and then discuss the quality of our real interest rate gap series, in particular its forecasting abilities. The rest of the paper is organized as follows. In Section 2 we provide some definitions of the neutral rate and briefly describe the approaches used to estimate it. In Sections 3 and 4, we report our estimation results based on the two methods used. In section 5, we assess the usefulness of these estimates for monetary policy. In particular, we test our estimates for leading indicator properties with regard to output and inflation. In addition, we simulate the path of interest rate using our estimates of the equilibrium rate and simple policy rules and compare the level of interest rate prescribed by the rule with the actual interest rate. Finally, we discuss the implications of uncertainties in estimating the neutral rate and data revisions for the use of the neutral rate for monetary policy. Section 6 provides some concluding remarks.

2. What is the neutral rate and how can we measure it?

2.1 What is the neutral rate?

Knut Wicksell provided the following definition for the neutral or natural rate:

4. In this paper, we use the terms neutral, natural or equilibrium interest rate interchangeably.
5. The 2002 BIS annual report contains explicit estimates of the neutral rate for several countries. In it, it is argued that the neutral interest rate can give monetary policy a firmer anchor, thereby reducing the risk of a central bank conducting monetary policy in a purely discretionary fashion.

"There is a certain rate of interest which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. The rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest on capital" (Wicksell 1898 [1936], p.102)

There appears to be two definitions. The first definition is in terms of the price level (or the rate of inflation). The second definition defines the neutral rate as the interest rate where the downward-sloping IS curve intersects a vertical line at potential output. If changes in prices are caused by an imbalance of aggregate demand and supply, then the two definitions are equivalent. In the literature, the neutral real interest rate is often defined as the real short-term interest rate which is consistent with output at its potential level and a stable rate of inflation.

The neutral interest rate is thus the level of interest rates that would prevail after ongoing temporary imbalances in the economy have worked themselves through. If the real interest rate is below the neutral rate, output will exceed potential, leading to higher inflation. If the real rate is above the neutral rate, inflation will eventually fall. While Wicksell's original writing is to relate the neutral rate to the price level, our definition is oriented towards the control of inflation, which seems appropriate given that the Bank of Canada aims at keeping inflation at 2 per cent, the mid-point of a target of 1 to 3 per cent.[6]

Under this definition, the neutral rate is not affected by short-term imbalances in the economy but rather by changes in the structure of the economy such as changes in the rate of population growth, the rate of technological change and the rate of time preference. As such, the neutral rate is not a fixed number and tends to vary as the economy is subject to shocks.

2.2 How can we measure the neutral rate?

There are two important characteristics about the neutral rate. First, it is unobservable and thus needs to be estimated, usually with a lot of uncertainty. Second, it is not a constant and tends to vary over time. Wicksell himself provides some insight as to how the neutral rate can be

6. Clinton (2004) argues that there are strong parallels between monetary policy in Canada and the writings of Wicksell.

estimated, by relating the price level (the rate of inflation) to the deviation of the actual interest rate from the neutral interest rate.

"For the current level of commodity prices provides a reliable test of the agreement or diversion of the two rates." (Wicksell, p.189)

Another straightforward way to calculate the neutral rate is to take the average of ex post real interest rate over a very long period, say 30 to 50 years. It is asserted that over such a long horizon, inflation is rather stable and the output gap is closed on average, when all markets are allowed sufficient time to clear and all variables in the economy grow at a constant rate. The resulting neutral rate, equal to the real interest rate at the long-run steady state equilibrium, can be interpreted as a long-run concept. However, such a long time horizon seems to be too far removed to matter for most policymakers. Besides, the factors influencing the neutral rate, say the fiscal balance, vary over time, and long historical averages may give a misleading impression of the current level.

Given the advance of economic theory and quantitative techniques, many researchers today, however, are prepared to estimate the neutral rate with sophisticated models and techniques. Several methods have been explored to estimate the natural rate of interest.[7] It is convenient to group the methods used in the literature to estimate the equilibrium real interest rates into categories.

At one end of the spectrum, estimates of the neutral rate can be derived from a well-articulated model, especially a dynamic stochastic general equilibrium (DSGE) model. First, a DSGE model with sticky prices that can match certain stylized facts is constructed. Second, the neutral interest rate is obtained by simulating the fully-flexible price equilibrium outcome under various real shocks, such as technology, preferences and government spending. When prices are flexible, output is at potential and thus the interest rate is at neutral. In other words, in this approach, the neutral rate can be defined as the real interest rate that would prevail if output is at potential and prices are flexible at each period in time (Woodford 2003). Using the model as a guide, one can construct empirical estimates of the neutral rate.

7. See Bomfin (2001) for a more complete review.

One advantage of a model-based approach is that the researcher can identify the sources of movements in the neutral rates as these estimates can be related to the deep structural parameters and the fundamental shocks of the model. However, estimates based on this approach usually are quite volatile, since the shocks buffeting the economy can move the neutral rate around. Policymakers who attempt to set interest rates at neutral at each period in time could generate excessive volatility in interest rates. Indeed, policymakers may not wish to keep rates at its neutral level every period, given that the economy is constantly subject to shocks which may be difficult to identify contemporaneously. Given the high uncertainty surrounding the estimates for the potential output and the neutral rate, it is undesirable to think of the neutral rate at such a high frequency. Moreover, estimates based on a model can very much depend on the assumptions underpinning the model and can thus depend on the specifications of parameters and shocks. In other words, they can be model specific.

At the other end of the spectrum, the neutral rate can be derived by applying simple statistical and/or filtering techniques, such as a multivariate filter, linear detrending and moving averages. A convenient proxy for the equilibrium real short-term interest rate is the real long-term interest rates. The long-term interest rate is believed to represent economic agents forecast of future short term interest rates, and implicitly a notional value of equilibrium interest rates. The difference between the real short and long rate—the term spread—is often used as a measure of stance of monetary policy.

While estimates based on purely statistical techniques are more straightforward to compute, they lack structural interpretation and may not be very useful in a policy context. For example, the use of the long-term interest rate as a proxy for the equilibrium interest rate requires a careful analysis of the sources of the movements in long-term rates. Moreover, when using the term spread, the risk premium is usually assumed to be constant over time. This assumption is not very realistic and if violated, makes the interpretation of the yield curve more complex.

Laubach and Williams (2001) take on a modelling approach that is somewhat between those two extremes, allowing the dynamics of the model to determine the time when the output gap is closed and the interest rate is at its neutral level. They construct a simple reduced-form model, consisting mainly of an IS curve and a backward-looking Phillips curve, which requires the real interest rate to equal the neutral rate when the output gap is closed and inflation is stable at its target. The IS curve relates the output gap to its own lags and the lagged real interest rate gap, defined as the difference between the observed real rate and the neutral rate. The Phillips curve specifies that the inflation rate is a function of its own lags, the output gap and a number of other variables. They also postulate that the neutral rate is proportional to trend output growth and a stationary, unobserved, stochastic component. Although the neutral rate is unobserved, once the model parameters are estimated, one can back out a series of the neutral rate estimates (together with the potential output and trend growth) for each period. This approach provides a neutral rate concept that focuses on the medium-run, when ongoing temporary imbalances in the economy have worked themselves out, and yet durable shocks, such as rising government debts, can change the neutral rate.

In sum, each of the approaches discussed above has its strengths and weaknesses. In this paper, we consider some of these approaches to estimate the neutral interest rate in Canada. First, we follow Laubach and Williams and employ a small-scale macroeconomic model consisting essentially of an IS and a Phillips curve to estimate the time varying neutral interest rate for Canada. Second, we follow Neiss and Nelson (2001) to calibrate a DSGE model with sticky prices.

3. Estimating a small model with Kalman filter

3.1. The basic model

Similar to Laubach and Williams, the model consists of the following set of equations:[8]

$$\tilde{y}_t = \phi_1 \tilde{y}_{t-1} + \phi_2 \tilde{y}_{t-2} + \phi_3 \sum_{i=1}^{2} [i_{t-i} - \pi_{t-i} - \bar{r}_{t-i}] + \varepsilon_{1t} \tag{3.1}$$

$$\pi_t = \theta_1 \pi_{t-1} + \theta_2 \pi_{t-2} + \theta_3 \pi_{t-3} + \theta_4 \tilde{y}_t + \theta_5 \Delta e_{t-1} + \varepsilon_{5t} \tag{3.2}$$

$$\bar{y}_t = \bar{y}_{t-1} + g_{t-1} + \varepsilon_{2t} \tag{3.3}$$

$$g_t = g_{t-1} + \varepsilon_{3t} \tag{3.4}$$

$$\bar{r}_t = \alpha g_t + z_t \tag{3.5}$$

8. Since we essentially employ their methodology, we refer the reader to their paper for more details.

$$z_t = \varphi_1 z_{t-1} + \varphi_2 z_{t-2} + \varepsilon_{4t} \tag{3.6}$$

Equation (3.1) is an IS function. In this equation, the output gap at time t depends on its own lags and on the lagged real interest rate; $\tilde{y}_t = (y_t - \bar{y}_t)$ is the output gap, y_t is the log of output, $\bar{y}_t$ is the log of potential output, i_t is the overnight rate, $\bar{r}_t$ is the unobserved natural rate of interest and $[i_{t-i} - \pi_{t-i} - \bar{r}_{t-i}]$ is the real interest rate gap. As in Kichian (1999), we assume that the output gap in Canada is characterized by an AR(2) process. There are two lags for the real rate gap and these coefficients are restricted to be equal. When the output gap is closed, the real interest rate will equal the neutral real rate. In the IS equation, transitory shocks and the short-run dynamics of the output gap are captured by the error term and the lags in the output gap while low frequency changes are captured by movements in the natural rate.

Equation (3.2) is a backward-looking Phillips curve. In this equation, π_t is the consumer price index, Δe_t is the first difference in the log of the real exchange rate and π_t^{us} is U.S. inflation.[9] Current inflation is a function of its own lags, lagged output gap and the lagged change in the exchange rate and lagged US inflation. We included three lags of inflation and also impose the restriction that the coefficients on the lags of inflation sum to 1. This ensures that there is no long-run trade-off between output and inflation. The interest rate gap affect inflation indirectly through its effect on the output gap.

In equations (3.3) and (3.4), we assume respectively that the natural level of output and its trend component evolve according to a random walk as in Gerlach and Smets (1997) and Laubach and Williams. Moreover, we impose some theoretical priors on movements of the natural rate of interest and assume that the natural rate of interest is related to the trend growth rate as in equation (3.5). We further assume that z_t, the unobserved component of the natural rate of interest is stationary and follows an AR(2) process as in equation (3.6).

3.2. Data, Estimation and Results

We estimate the simple structural Canadian model using quarterly Canadian data over the period 1965Q1 to 2003Q4. The estimation consists of obtaining estimates of the parameters $\{ \phi_1, \phi_2, \phi_3, \theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \alpha, \varphi_1, \varphi_2 \}$ and the standard errors of $\{\varepsilon_i\}$, $i = 1, \ldots, 4$.

9. We use total CPI for Canada.

Equations (3.1) and (3.6) can be written in a state-space form, with the equations (3.1) and (3.2) making up the measurement equations while the rest being the transition equations.

The estimation methodology consists essentially of two stages.[10] In the first step, output gap estimates are generated using Kuttner's (1994) approach which consists of applying the Kalman filter to equation (1) but omitting the interest rate gap from that equation and assuming that the trend growth rate is constant. Conditional on these preliminary estimates of the output gap, we then estimate the remaining parameters of the model by maximum likelihood method using standard Kalman filtering techniques. The unobserved component is subsequently obtained by filtering.

The results from maximum likelihood estimation of the model are reported in Table 1. We find that the two lags on the output gap in the IS function are significant and sum to less than one, indicating that this equation is stable. The real interest rate enters the IS with the correct negative sign and is significant at the 5% level. On the other hand, we had more problems finding significance for all the coefficients in the Phillips curve. For example, the coefficients on U.S. inflation and the log change in the real exchange rate are not significant at the 5% level. However, adding these variables helped our model converge. This indicates that our estimates may be heavily dependent on the structure of our model, a criticism that regularly applies to state-space models.

4. Estimation using a calibrated sticky-price model

4.1. A sticky-price model with capital adjustment costs

The model is the one described in Lam and Tkacz (2002), which is essentially the model in Neiss and Nelson (2000). This model is found to produce dynamic response functions to shocks that are consistent with observed responses and the neutral rate estimates derived from this model contains useful information about future economic activity. There are three agents - households, firms, and a monetary authority.[11]

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10. Given that the variance of potential growth and the neutral rate is considerably smaller than that in the data, estimates of the standard deviation of the trend growth rate and the unobserved natural interest rate are not statistically different from zero. Stock and Watson's median unbiased estimator is used to calculate the signal-to-noise ratio, which is then imposed when estimating the model. Details of the estimation methodology can be found in Laubach and Williams (2001).
11. We briefly describe the model here and we refer readers to Neiss and Nelson as well as Lam and Tkacz for a more detailed description of the model set up as well as the first-order conditions.

4.1.1. Households

The economy is composed of a continuum of infinitely-lived agents where each of them consumes a final good (C_t), hold real money balances $\left(\frac{M_t}{P_t}\right)$, and supplies labour (N_t). They are assumed to maximize the sum of discounted expected utility by choosing the optimal quantity of goods to consume and the amount of hours to work and invest in physical capital (K_{t+1}) each period given prices (P_t), wages (W_t) and interest rates (r_t). The representative household chooses a sequence of consumption, nominal money balances, one period bond holdings (B_{t+i}), capital (K_{t+i}) and employment to maximize the following lifetime utility function:

$$\max E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\xi_t\frac{\sigma}{\sigma-1}\left(\frac{C_t}{C_{t-1}^h}\right)^{\frac{\sigma-1}{\sigma}}+\frac{a_m}{1-\gamma_m}\left(\frac{M_t}{P_t}\right)^{1-\gamma_m}+a_nL_t\right]\right\} \tag{4.1}$$

subject to a series of period budget constraints:

$$C_{t+i}+I_{t+i}+\frac{M_{t+1}}{P_{t+i}}+\frac{B_{t+i+1}}{P_{t+i}} = w_{t+i}N_{t+i}+r_{t+i}K_{t+i}+\Pi_{t+i}+\frac{M_{t-1+i}}{P_{t+i}}+\frac{(1+R_{t-1+i})B_{t+i}}{P_{t+i}}$$
$$-H(I_{t+i}) \qquad \forall i = 0, 1, \ldots, \infty \tag{4.2}$$

and $L_t+N_t\le 1$ (4.3)

where $I_{t+i} = K_{t+i+1}-(1-\delta)K_{t+i},\ \delta\in[0,1]$ (4.4)

$$H(I_{t+i}) = \psi I_{t+i}^{\eta},\ \psi>0,\ \eta>1 \tag{4.5}$$

and $\xi_t = \rho_\xi\xi_{t-1}+\varepsilon_t^{\xi}$ (4.6)

Consumption in the model displays habit formation since households derive utility from current but also from past consumption. The parameter h measures the strength of habit persistence. In the limiting case where $h = 0$, the function reduces to a standard time-separable utility function. The parameter σ measures the curvature of the consumption function and corresponds to the intertemporal elasticity of substitution and ξ_t represents a preference shock that evolves according to (4.6).[12]

In equation (4.2), r_t represents the rental rate of capital, R_t is the gross nominal interest rate and Π_t denotes lump sum firm profits. I_t denotes investment and it is related to the capital stock as in equation (4.4) where δ is the depreciation rate. We assume that it is costly for households to adjust their capital stock (equation (4.5)). With $\eta = 2$, equation (4.5) amounts to quadratic costs of adjustment.

4.1.2 Firms

There are two types of firms: final and intermediate goods-producing firms. We assume that there are a large number of final goods producers who behave competitively and who produce a homogenous good, Y_t, using intermediate goods, $Y_t(z)$. We also assume that there is a continuum of intermediate goods producers owned by consumers, indexed by the letter z who operate in a Dixit-Stiglitz style imperfectly competitive economy.

Final Goods Producers

Final goods producers use the following production function to transform intermediate goods into final output:

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{4.7}$$

where $\theta > 1$. In each period, the final goods firm choose inputs $Y_t(z)$ for all $z \in [0, 1]$, and output Y_t to maximize profits subject to the production function and price P_t specified in equation (4.7).

Intermediate Goods Producers

Each intermediate goods producer is indexed by $z \in [0, 1]$, operates in a Dixit-Stiglitz style imperfectly competitive economy and faces a downward sloping demand function for his

12. According to McCallum and Nelson (1999), this preference shock represents a demand shock in the expectational IS.

product. In addition, each firm produces intermediate goods subject to the following technology constraint:

$$Y_t(z) = A_t N_t(z)^{\alpha} K_t(z)^{1-\alpha} \tag{4.8}$$

where $N_t(z)$ and $K_t(z)$ are respectively the amount of labour and capital hired by the firm to produce output and A_t is a technology shock which follows this process:

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_t^A \text{ and } \varepsilon_t^A \sim iid(0, \sigma_A)$$

The intermediate goods producer is assumed to choose the optimal amount of physical capital and labour to maximize profits taking the productivity of the firm as given subject to equation (4.8).

Inflexible Prices

Intermediate goods producing firms set nominal prices on a staggered basis. We follow Calvo (1993) and assume that firms are allowed to reset the price of their good in any given period with probability $(1-s)$.[13] Hence they keep their price fixed with probability s. The parameter s governs the degree of nominal price rigidity in the model. As s approaches zero (one), prices become more flexible (rigid).[14]

Firms that are able to adjust their price will do so optimally and will maximize expected profits, taking aggregate output (Y_t), the aggregate price level (P_t), nominal marginal cost (MC_t) and the constraint on the frequency of price adjustment as given. Firms that are not able to change their price, adjust output to meet demand. However, all firms minimize their costs given demand.

The problem of the firm changing prices at time *t* consists of choosing $P_t(z)$ to max the following

13.An alternative is to follow Christiano, Eichenbaum and Evans (2001) who modify Calvo's framework by assuming a dynamic price-updating scheme instead of a static one. One advantage of their methodology and the assumption of dynamic price-updating is that the lagged inflation term can be derived in a non-trivial way without having recourse to some adhoc assumptions.

14.If firms face a probability $(1-s)$ of changing its price, prices will remain fixed on average for $\frac{1}{(1-s)}$.

$$E_t \sum_{i=0}^{\infty} (s\beta)^i \left[\Lambda_{t,t+i} \left(\frac{P_t(z) - MC_{t+i}}{P_{t+i}} \right) Y_{t,t+i}(z) \right] \tag{4.9}$$

subject to the demand for its good $Y_{t,t+i}(z) = \left[\frac{P_t(z)}{P_{t+i}}\right]^{-\theta} Y_{t+i}$ where $\Lambda_{t,t+i}$ is the ratio of the marginal utility of consumption at time $t+i$ and $t.\beta\Lambda_{t,t+i}$ is the rate at which firms discount earnings at time $t+i$.

The first order condition to this maximization problem can be written as

$$P_t(z) = \mu E_t \sum_{i=0}^{\infty} \omega_{t,t+i} MC_{t+i} \tag{4.10}$$

where $\omega_{t,t+i} = \dfrac{(s\beta)^i \Lambda_{t,t+i} D_{t,t+i}}{E_t \sum_{i=0}^{\infty} (s\beta)^i \Lambda_{t,t+i} D_{t,t+i}}$, $\mu = \dfrac{1}{1 - \frac{1}{\theta}}$ is the steady state mark up or the inverse of the steady-state real marginal cost and $D_{t,t+i} = \left[\frac{P_t(z)}{P_{t+i}}\right]^{1-\theta} Y_{t+i}$ denotes firm's revenues at $t+i$ conditional on $P_t(z)$ as in Gali and Gertler.

As firms know that they cannot change their prices in each period, their optimal price will be a function of past and expected future demand. As a result, the aggregate price index will be a weighted average of the optimal price and prices set in period *t*-1 to reflect the proportion of firms that are not able to change their prices. The aggregate price index can thus be expressed as

$$P_t^{1-\theta} = [(1-s)\tilde{P}_t^{1-\theta} + sP_{t-1}^{1-\theta}] \tag{4.11}$$

By log-linearizing the FOC of the firm and equation (4.11) above, we obtain a supply function which Roberts (1995) has labelled the "New Keynesian" Phillips curve. This supply function is given by

$$\pi_t = \beta E_t(\pi_{t+1}) + \varphi mc_t \tag{4.12}$$

where $\varphi = \left[\frac{s}{1-s}\right][1 - (1-s)\beta]$

For this "New Keynesian" Phillips curve to generate realistic inflation dynamics, we follow Gali and Gertler (1999) to assume that a proportion of firms use a simple rule of thumb that is based on the recent history of aggregate price behaviour to set prices. When this is combined with the model of Calvo, one obtains the following hybrid supply function:

$$\pi_t = \phi\pi_{t-1} + (1-\phi)E_t(\pi_{t+1}) + \varphi mc_t. \tag{4.13}$$

In equations (4.12) and (4.13), the parameter φ governs the degree to which prices are sticky. The larger (smaller) φ is, the more flexible (rigid) prices are. The equilibrium real rate is obtained by calibrating φ to a very large number while the observed (actual) rate is obtained by assuming that prices are rigid. Once the two measures are obtained, we can construct the interest rate gap, i.e., the difference between the equilibrium rate (flexible-price value) and the observed rate (sticky price value). While the specification of the supply function and interest rate setting rule does not affect our measure of the equilibrium rate, however, the observed or actual interest rate will be dependent on how the model is specified.

4.1.3 Monetary Authority

Monetary policy in the model is characterized by an interest rate rule. Following Clarida, Gali and Gertler (1998), we estimate a forward-looking rule for Canada for the period 1990Q1 to 2000Q4. This rule is given by:

$$R_t = 0.816R_{t-1} + (1-0.816)[2.075E_t\pi_{t+1} + 1.08(y_t - \bar{y}_t)] + \varepsilon_t^R \tag{4.14}$$

where ε_t^R is a policy shock and $(y_t - \bar{y}_t)$ is the output gap.

4.1.4. Market Clearing and Equilibrium

We define an equilibrium as a collection of allocation for final goods-producing firms, $Y_t, Y_t^D(z)$ for all $z \in [0, 1]$; allocations for intermediate goods producing firms, $N_t(z), K_t(z)$ for all $z \in [0, 1]$; allocations for consumers, $C_t, N_t, I_t, M_t, B_{t+1}, K_{t+1}$; together with a price vector $W_t, Z_t, P_t, P_t(z), R_{t+1}^n, MC_t, H(I_t)$ for all $z \in [0, 1]$ such that all agents are maximizing subject to the constraints they face, supply equals demand in each market and all resource constraints are satisfied, given the values of the predetermined and exogenous variables. To examine the dynamics of the model, we log-linearize the optimality conditions around their steady state.[15]

4.2. Calibration

In calibrating the model, we follow Dib (2002) and Dolar and Moran (2002) who have estimated several of these parameters for Canada. The aggregate supply curve is the hybrid function described in (4.14). Table 2 summarizes the parameter values set for the model.

4.3. Empirical Estimate of the Equilibrium Real Rate.

As explained in Neiss and Nelson, the neutral interest rate in such a model can be expressed as a linear function of the technology and preference shocks. Then using data on preference and technology shocks and an equation that links the neutral rate to these shocks, one can construct an empirical series for the equilibrium rate. The methodology is summarized below.

1. We first solve the model under flexible prices and then using stochastic simulations, generate artificial data for the interest rates and the real innovations. Neiss and Nelson argues that the equilibrium interest rate can be written as a linear combination of current and past dated shocks. Thus using the artificial data generated from each stochastic simulation, we run the following regressions for an *i* that is high enough for a good fit.[16]

$$r_t^* = \Phi_1 \xi_t + \Phi_2 \xi_{t-1} + \ldots + \Phi_i \xi_{t-i} + \Upsilon_1 a_t + \Upsilon_2 a_{t-1} + \ldots \Upsilon_i a_{t-i} \tag{4.15}$$

We then calculate the average of each of the above coefficients across the simulations, which will be used below to generate our model-based and empirical series for the equilibrium real rate.

2. In most models, technology shocks are measured by Solow residuals which are usually obtained by subtracting the log values of labour and capital inputs, weighted accordingly, from the log of total output. In this paper, we however use the Solow residuals series from QPM, the Bank of Canada's main projection model.[17]

3. To construct the preference shocks, we proceed as follows: Using the log-linearised equations for consumption, bonds and the law of motion for preference shocks, we derive the following equation which relates preference shocks to consumption, prices and interest rates:[18]

15.See Lam and Tkacz (2002) for the log-linearized equations.
16.The fit of these equation is very good, implying that our regression approximates well the true measure of the equilibrium rates.
17.As our model does not allow for sectoral growth, the series for the Solow residuals in our model corresponds to detrended total factor productivity.
18.(See Appendix II in Lam and Tkacz for more details.

$$\xi_t = \frac{1}{\upsilon_6}[\upsilon_2 \Delta c_t + \upsilon_3 E_t \Delta c_{t+2} - \upsilon_4 E_t \Delta c_{t+1} - \upsilon_5 (R_t - E_t \pi_{t+1})] \quad (4.16)$$

In the above equation, the preference shock at time *t*, ξ_t, is related to Δc_t, and exante real interest rate. To calculate ξ_t, we use c_t the log of consumption excluding durables, π_t, annual core inflation, and R_t is the 90-day commercial paper rate, where the $\upsilon's$ are determined by the model's parameters. To generate expected values for consumption and inflation, we use a 3-variable VAR estimated over the period 1980q4 to 2004q2.[19] Using our calibrated values, the coefficients estimated in equation (4.15), the technology shock and preference shock series that we constructed, we can obtain an empirical series for the equilibrium real rate. Our series spans from 1985q2 to 2004q2.

4. The real neutral interest rate is given by[20]

$$r_t{}^* = -0.0052\xi_t + 0.011\xi_{t-1} + 0.0034\xi_{t-2} + 0.0009\xi_{t-3} + 0.0001\xi_{t-4} - 0.0001\xi_{t-5}$$
$$-0.0151a_t + 0.0094a_{t-1} + 0.0019a_{t-2} - 0.0005a_{t-3} - 0.0011a_{t-4} - 0.0014a_{t-5} \quad (4.17)$$

5 Assessing the estimated neutral rates

We have presented estimates of the neutral rates based on several methods commonly used in the literature. To assess how useful these estimates are for monetary policy, we first examine their leading indicator properties for future output growth and inflation. As such, we can evaluate whether the real rate gap can be used as a good measure of monetary policy stance. Second, we use the time-varying neutral rate measures to back out the interest rates prescribed by a Taylor-type policy rule in order to assess past policy actions. Finally, we discuss the implications on our estimates of uncertainty regarding the estimation method and data revisions.

Before proceeding to our assessments of the estimated neutral rates, we discuss some general properties of these estimates. The neutral rate estimates for Canada from the methods proposed by Laubach and Williams (2003, LW hereafter) and Neiss and Nelson (2003, DSGE hereafter), are reported in Figures 1. We include two estimates for the LW method: the two-sided

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19.The VAR contains ten-lags of the following variables, $\Delta c_t, \pi_t, \Delta R_t$ with a dummy at 1992q1 to capture changes in the monetary regime.

20.We have omitted low coefficients.

estimates (LW2) allows for smoothing while the one-sided series (LW1) does not.[21] The three series in Figures 1 follow a broadly similar pattern, with a number of exceptions. First, the estimates from the DSGE model display higher volatility, ranging from just below zero to about 6% in 1985-2004, since the series follow the ex post real interest rates rather closely. LW1 and LW2, however, have a narrower range of 2.5-4.5% over the same period. Second, the DSGE estimates climbed to 6% around 1990, a period burdened with fiscal deficits and when the Bank was trying to bring inflation down, while the other two estimates were only about 4%. Third, since 2001, the DSGE estimates have been very low, averaging to about 1%, which could be attributed to the healthy fiscal balances, reduced inflation uncertainty and lower risk premiums. In sum, these results suggest that both the level and the volatility of the neutral rate estimates have declined over time, consistent with findings in other developed countries.[22] Figure 3 shows the real interest rate gap coming from the two models and the term spread.

5.1. In and out-of-sample forecasts

Here we assess how well our different neutral rate estimates perform in forecasting GDP growth and inflation in Canada. If the neutral rate is a good measure of the stance of monetary policy, it should provide information about future GDP growth and inflation. Figure 4 suggests that the real rate gap tends to lead output growth (positive real rate gap implies tight policy). Since the term spread is found to be a good measure of policy stance and a good predictor of future economic activity (see e.g. Cozier and Tkacz), we compare our results to that of the term spread. First, we construct the real interest rate gap, defined as the difference between the actual real interest rate and the derived neutral rate. Then we assess the information content of the real interest rate gap and the term spread for output growth and inflation, over the 4- and 8- quarter horizon. Both in-sample and out-of-sample analysis will be conducted.

Many approaches could be used to assess the information content of variables. We follow the methodology of Stock and Watson(2003), which is based on the following equation:

$$\Delta y^h_{t+h} = c + \alpha(L)\Delta y_t + \beta(L)x_t + \varepsilon^h_t \qquad (5.\ 1)$$

21. Smoothed estimates means that the estimated value at date t is based on the entire sample, not just the data from dates s<t.

22.See for example, Marques and Manrique (2004) for Germany and the United States and Basdevant, Bjorksten and Karagedikli (2003) for New Zealand.

where:

$\alpha(L)$ and $\beta(L)$ are polynomial lag operators[23];

$\Delta y_t = (y_t - y_{t-4})$;

$\Delta y^h_{t+h} = (y_{t+h} - y_t)$ is the variable to predict h periods ahead; and

$x_{t:}$ is the variable to be evaluated.

If the variable to be forecast is the percentage growth of GDP eight quarters ahead, then $\Delta y^8_{t+8} = 50[\log(GDP_{t+8}) - \log(GDP_t)]$, calculated at an annualized percentage growth rate. Since data is overlapping in this model, we calculate a consistent covariance matrix allowing for heteroscedasticity and serial correlation by applying the White (1980) correction to the variance-covariance matrix of the residuals.[24] The estimation period runs from 1985Q1 to 2003Q4. For GDP growth, we assess the information content four periods ahead while for inflation and core inflation, estimations are done for both the four and eight quarters horizons. These lag lengths are chosen since monetary policy is believed to affect GDP growth and inflation with a four and eight quarters lag, respectively.

We consider two specifications. In the non-autoregressive model, we suppose $\alpha(L) = 0$ while in the autoregressive model, we consider $\alpha(L) \neq 0$. The latter specification helps address mispecification problems that may exist.

The in-sample evaluation is based on two criteria: 1) the expected sign of the coefficient of the *x* variable and its statistical significance; 2) the adjusted $\bar{R}^2$. The out-of-sample evaluation is based on the root mean squared error (RMSE) of the forecasts, the mean error (ME), the mean absolute error (MAE) and the confusion rate (CR). While the RMSE, ME, MAE are more commonly known, it is useful to provide a brief description of the CR. The CR is a timing criterion used to forecast turning point, retrieved from a two-by-two contingency table. It allows us to calculate the frequency of cases wrongly predicted by the model. As such, the best model would be the one that have the lowest value of the CR. Overall, the variable that gives the smallest values for all out-of-sample indicators (RMSE, ME, MAE and CR) is the best leading indicator.

23. We used one lag.
24. We use the command ROBUSTERRORS in RATS for computing a consistent estimate of the covariance matrix (specifying 3 lags in the MA process).

Where necessary, a loss differential test of Diebold and Mariano (1995) is applied. This test examines whether two models are statistically equally accurate, based on predictive ability.

While there is a general consensus on the leading indicator properties of the term spread (short-term interest rate minus long-term rate) for future economic activity, the theoretical foundation is rather weak (for example, Rendu de Lint er al., 2003). It is generally found that larger (smaller) spreads lead periods of stronger (weaker) future economic activity. It is, however assumed that both the short and the long term interest rates have an equal but opposite effect on future economic activity. There is no strong reason to believe that the economy should react in the same way to the movements in the long and short term interest rate. Besides, not only the spread matters for future economic activity, but also the level of the interest rates. A term spread equal to a hundred basis points when the average interest rates is 4% could have a different impact on GDP growth than when the average interest rates is 8%. To account for this possibility, on top of the regular spreads and gaps, we consider in our analysis cases where both variables (short and long run measures) enter the regressions separately.

$$\Delta y^{h}{}_{t+h} = c + \alpha(L)\Delta y_t + \beta(L) short_t + \gamma(L) long_t + \varepsilon^{h}{}_t \qquad (5.\ 2)$$

Finally, since the neutral interest rate should be seen as a medium-run concept, we use a policy rule to infer the corresponding short-run interest rates. The policy rule used in the analysis is from Côté et al. (2002), which is a Taylor-rule with a larger coefficient on the inflation gap. This rule is found to be empirically robust across a number of models for Canada.

$$r_{s,t} = \bar{r} + \bar{\pi} + 2(\pi_t - \bar{\pi}) + 0.5 ygap \qquad (5.\ 3)$$

where:

$\bar{r}$: is the neutral interest rate;

$\bar{\pi}$: is the inflation target (2%);

π_τ: is the observed inflation;

ygap: is the output gap.

We apply the previous tests to the short term interest rate measures derived from the policy rule in equation (5.3). The results for all the measures and combinations are reported in Tables 3 to 7. The variables used in the assessment are described as follows:

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R10-R90	Term spread;
R10	Real 10 years government bond rate, deflated by total inflation;
R90	Real 90 days commercial paper, deflated by total inflation;
dgerstar	Real neutral rate coming from the DSGE model;
rstar1sided	Real neutral rate coming from the one-sided Kalman filter model;
rstar2sided	Real neutral rate coming from the two-sided Kalman filter model;
x1 (dgerstar)	The short term interest rate suggested by equation (5.3) when a neutral rate from the DSGE model is used as an equilibrium real rate;
x2(rstar1sided)	The short term interest rate suggested by equation (5.3) when a neutral rate from the Kalman Filter model is used as an equilibrium real rate;
x3(rstar2sided)	The short term interest rate suggested by equation (5.3) when a neutral rate from the Kalman Filter model is used as an equilibrium real rate.

5.1.1. In-sample results

Tables 3 and 7 report the in-sample results. The second column in Table 3 shows the results for GDP growth. All the coefficients for the different measures of interest rate gaps have the correct sign and are mostly statistically significant. Based on the $\overline{R}^2$, with and without an autoregressive component, we find that most of the interest rate gap measures perform as well as the term spread. The improvement is substantial for certain measures For example, the most marked improvement for the in-sample fit occurs in the case with the gap using the one-sided neutral rate measure plus the square of the interest rate gap.

We also assess the information content for total and core inflation. The results are reported in Tables 3 to 6. All the coefficients on the stance variables of the correct sign and mostly significant. Based on the $\overline{R}^2$ measure, the neutral rate measures outperform the term spread at four and eight quarters horizon in the autoregressive model. The results for core inflation are similar. For the autogressive version, certain neutral rate measures can outperform the term spread at four and eight quarters horizon in forecasting total inflation.

In sum, based on the $\overline{R}^2$ measure, the interest rate gap can improve the in-sample fit of all equations with respect to GDP growth and inflation, at different horizons.

5.1.2. Out-of-sample results

The out-of-sample forecasts are calculated using rolling regressions from 1993:Q1 to 2003:Q4. Again, we calculate for every variable or combination of variables the mean error (ME), the mean absolute error (MAE), the root mean square error (RMSE) and the confusion rate (CR). The results are reported in Tables 7 to 8, for both the non-autoregressive and autoregressive models.

For GDP growth, the interest rate gaps using LW1 or LW2 plus its squared value give the lowest RMSE. The combination of the one-sided measure and its squared variable gives the lowest RMSE for the autoregressive model. Indeed, the one-sided neutral rate in deviation from real 90 days interest rate and its squared variable, gives lower RMSE for both models, on average, even if it is not the lowest in every model. In general, the neutral rate measures give lower RMSE than the term spread measure. However, the ME is lower for the term spread while the CR is comparable to the other variables.

With respect to inflation, all the interest rate gap measures perform as well as the term spread. The RMSE improves in certain cases, in particular in the autoregressive model.

5.2 Equilibrium interest rate and implied policy rule.

Figure 4 shows the different measures of the implied policy rate (*x1, x2,x3*) and the actual overnight rate from 1985 to 2003. It appears that the three neutral rate measures move together closely and track the actual real rate well. In general, the correlation between them is very high. Its is interesting to note that starting from 1985 and up to 1990, the observed policy rate and the implied short rate measures (*x1, x2, x3*) move very closely together. From 1991 up to 1999, the implied measures seem to be below the historical policy rate. Between 2002 and 2003, the rule seems to suggest a higher short term interest rate compared to the observed policy rate. Apart from the period 1985 to 1990, the interest rates implied by the rule is not close to the historical interest rate. For the period 1991-1999, the recommendation coming from the rule have likely understated the short term interest rate because it did not take into account the high public debt and the implied high risk premia.

5.3. Uncertainties and performance of the neutral rate

There are two types of uncertainties that we would like to address in this paper. The first one is related to the estimation method used to obtain the neutral rate measures. The second stems from data revisions of GDP and the output gap. Both uncertainties should impact on the performance of the neutral rate measures compared to the TS and have important implications on the using the neutral rate for monetary policy.

5.3.1. Uncertainty related to the estimation

In order to assess the uncertainty regarding estimation on the neutral rate measures, we calculate the confidence bands for the one sided and two sided neutral rate measures. The results reported in Figures 5 and 6 suggest that the confidence bands are wide. While the standard deviation is time varying, we just considered the mean of the standard deviation over the historical sample to construct them. In order to assess the effect of this uncertainty on the leading information properties of the neutral rates measures, we choose one of the combination variable (x3, R90) that gives one of the best RMSE (0.52) compared to the TS (1.02), for inflation at eight quarters ahead. Over 1985Q1 to 2003Q4 and we generate for every observation a measure of the two-sided neutral interest rate as described below.

At every point in time, we draw a number from a uniform distribution over the range between -2 and 2. Then we add to the observed level of the neutral rate (one-sided or two-sided) the value of the number drawn, from the uniform distribution, times the standard deviation of the real neutral interest rate. Given the values from equation 5.3, we generate the implied path of policy level implied by our rule. And given the path of x3 and R90, we estimate equation (5.1) rolling over from 1993Q1 to 2003Q4 at eight quarters horizon. We calculate the RMSE implied by the latest calculated vector. We repeat this exercise 1000 times to generate 1000 different values for the RMSE and then compare them to the RMSE coming from the forecast using the term spread. The results are reported in Figure 10. Over all the simulation, the average value of the RMSE for all the different *rstar1sided* is equal to 1.02. The probability that the RMSE(two-sided) is higher than the RMSE(TS) is equal to 0.001 (one out of one thousand times). So even when we account for the uncertainty coming from the estimation, the neutral rate measures still show better good leading information properties for the economic variables.

5.3.2. Uncertainty related to data revisions

Data revisions occur frequently, especially for GDP and the output gap. While the data is refined as more information is available, monetary policy actions have to be taken on a real-time basis. However, when the data is revised, this may impact on the real time performance of some indicators compared to others. While the term spread is not subject to revisions, the neutral rate interest rates and related short term interest rates are likely to be revised because of revisions in the variables used in the underlying models. To asses the effect of data revisions on the neutral rate estimates, we used the following methodology. First we take the latest GDP growth series available. For every period in time, we compare it to the first release of the GDP growth data.[25] We calculate over history the standard deviation of revisions. Using this standard deviation, we build the upper band and lower band for the GDP series in level, given the latest data series (Figure 7). Using those bands, we run the Kalman filter model and extract the respective one-sided (Figure 8) and two-sided estimates (Figure 9). The last two figures show clearly that data revision have very minor impact on the level of the neutral rate measures. And as long as the uncertainty is time invariant, there is no effect on the forecasting abilities of the neutral rates.

6. Conclusions

In this paper, we present several estimates of the neutral interest rate in Canada based on the methodology in Laubach and Williams as well as Neiss and Nelson. Using these estimates, we construct real interest rates gaps as measures of the stance of monetary policy. The information contents of these stance measures are then examined. Our results indicate that the real interest rate gap is at least as good a predictor for future economic activity as the yield spread. On this basis, the real interest rate gap appears to be a useful tool for policymakers.

While there is a high degree of uncertainty regarding the estimation of the neutral rate, our estimates of the neutral rate seem to be quite useful as an indicator of economic activity. However, more work needs to be done to improve our measures and our understanding of the forces that are influencing the neutral rate.

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25. This exercise does not take into account the different vintages of data. We are collecting vintage data on GDP and we plan to calculate the range of neutral rates as a result of data revisions as in Kozichi(2004).

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Table 1: : Maximum Likelihood estimates

Regressor	Parameter	Baseline
$\tilde{y}_{t-1}$	ϕ_1	1.59 (11.4)
$\tilde{y}_{t-2}$	ϕ_2	-0.64 (4.64)
$\tilde{y}_{t-1} + \tilde{y}_{t-2}$	$\phi_1 + \phi_2$	0.95
r_{t-1}	ϕ_3	-0.09 (3.15)
π_{t-1}	θ_1	1.29 (17.8)
π_{t-2}	θ_2	-0.19 (1.48)
$\tilde{y}_{t-1}$	θ_4	-0.04 (0.7)
Δe_{t-1}	θ_5	-0.0088 (0.60)
π^{us}_{t-1}	θ_6	0.99 (1.76)
s.e ($\bar{r}$)	2.82	
s.e ($\bar{y}$)	4.50	
s.e (g)	0.54	

Table 2: Parameter Values for the DSGE Model

Parameters	Description	Value
β	Discount factor	0.99
δ	Depreciation rate	0.025
α	Share of labour	0.7
h	Habit persistence	0.6
μ	Steady-state mark-up	1.25
ψ	Adjustment cost parameter	0.2
η	Adjustment cost parameter	2
s	probability of changing prices	0.25
σ	Intertemporal elasticity of substitution	0.6
ρ_ξ	Autocorrelation of preference shock	0.985
σ_{ε^ξ}	Std deviation of preference shock	0.015
ρ_A	Autocorrelation of technology shock	0.95
σ_{ε^A}	Std deviation of technology shock	0.01

Table 3: In-Sample estimation, four quarters ahead non-autoregressive model

	GDP growth		Total inflation		Core inflation	
Variables	Coefficients and P-values	$\bar{R}^2$	Coefficients and P-values	$\bar{R}^2$	Coefficients and P-values	$\bar{R}^2$
R10 - R90	0.72(0.00)	0.29	-0.59(0.00)	0.36	-0.21(0.00)	0.10
R90 - dgerstar	-0.35(0.28)	0.02	0.50(0.03)	0.11	0.19(0.11)	0.03
R90 - rstar1sided	-0.50(0.03)	0.22	0.32(0.03)	0.17	0.12(0.04)	0.05
R90 - rstar2sided	-0.62(0.00)	0.31	0.30(0.06)	0.12	0.09(0.23)	0.01
R10 - r90, (R10 - R90)2	0.93(0.00) -0.25(0.00)	0.43	-0.66(0.00) 0.09(0.19)	0.39	-0.21(0.00) 0.01(0.89)	0.09
R90 - dgerstar, (R90 - dgerstar)2	1.04(0.03) -0.41(0.00)	0.10	0.65(0.08) -0.04(0.72)	0.10	0.38(0.17) -0.06(0.44)	0.02
R90 - rstar1sided, (R90 -rstar1sided)2	0.35(0.01) -0.20(0.00)	0.49	-0.03(0.87) 0.08(0.01)	0.25	0.11(0.32) 0.00(0.92)	0.03
R90 - rstar2sided, (R90 - rstar2sided)2	0.23(0.22) -0.19(0.00)	0.49	-0.10(0.64) 0.09(0.02)	0.19	0.08(0.52) 0.00(0.96)	-0.00
R90 - dgerstar, R90 - rstar1sided	1.12(0.00) -0.99(0.00)	0.33	0.05(0.88) 0.30(0.21)	0.16	0.02(0.93) 0.11(0.30)	0.03
R90 - dgerstar, R90 - rstar2sided	1.22(0.00) -1.15(0.00)	0.47	0.24(0.47) 0.19(0.45)	0.12	0.18(0.44) 0.01(0.96)	0.01

Table 4: In-Sample estimation, eight quarters ahead non-autoregressive model

	Total inflation		Core inflation	
Variables	Coefficients and P-values	$\bar{R}^2$	Coefficients and P-values	$\bar{R}^2$
R10 - R90	-0.52(0.00)	0.36	-0.18(0.02)	0.06
R90 - dgerstar	0.32(0.12)	0.04	0.10(0.40)	-0.00
R90 - rstar1sided	0.25(0.04)	0.10	0.10(0.18)	0.02
R90 - rstar2sided	0.16(0.24)	0.03	0.02(0.80)	-0.01
R10 - r90, (R10 - R90)2	-0.51(0.00) -0.05(0.32)	0.37	-0.14(0.03) -0.07(0.17)	0.09
R90 - dgerstar, (R90 - dgerstar)2	0.82(0.11) -0.14(0.26)	0.05	0.53(0.13) -0.12(0.17)	0.01
R90 - rstar1sided, (R90 -rstar1sided)2	0.22(0.38) 0.01(0.87)	0.08	0.31(0.12) -0.04(0.23)	0.03
R90 - rstar2sided, (R90 - rstar2sided)2	0.12(0.70) 0.01(0.86)	0.01	0.21(0.34) -0.04(0.31)	-0.01
R90 - dgerstar, R90 - rstar1sided	-0.04(0.88) 0.27(0.12)	0.08	-0.08(0.69) 0.14(0.30)	0.00
R90 - dgerstar, R90 - rstar2sided	0.25(0.34) 0.05(0.79)	0.03	0.16(0.46) -0.05(0.74)	-0.02

Table 5: In-Sample estimation, four quarters ahead autoregressive model

	GDP growth		Total inflation		Core inflation	
Variables	Coefficients and P-values	$\overline{R}^2$	Coefficients and P-values	$\overline{R}^2$	Coefficients and P-values	$\overline{R}^2$
R10 - R90	0.67(0.00)	0.39	-0.47(0.01)	0.37	0.04(0.53)	0.66
R90 - dgerstar	-0.42(0.18)	0.16	0.67(0.00)	0.45	0.11(0.05)	0.68
R90 - rstar1sided	-0.44(0.03)	0.31	0.28(0.02)	0.36	0.02(0.64)	0.66
R90 - rstar2sided	-0.54(0.00)	0.37	0.27(0.02)	0.34	0.01(0.85)	0.66
R10 - r90, (R10 - R90)2	0.85(0.00) -0.21(0.01)	0.49	-0.56(0.00) 0.11(0.10)	0.41	0.04(0.43) -0.01(0.81)	0.66
R90 - dgerstar, (R90 - dgerstar)2	0.92(0.04) -0.40(0.00)	0.25	0.68(0.00) -0.00(0.98)	0.45	0.12(0.32) -0.00(0.93)	0.67
R90 - rstar1sided, (R90 -rstar1sided)2	0.32(0.02) -0.18(0.00)	0.54	-0.03(0.84) 0.07(0.03)	0.42	0.07(0.33) -0.01(0.36)	0.66
R90 - rstar2sided, (R90 - rstar2sided)2	0.24(0.21) -0.18(0.00)	0.53	-0.07(0.75) 0.08(0.07)	0.39	0.08(0.35) -0.02(0.30)	0.66
R90 - dgerstar, R90 - rstar1sided	0.99(0.01) -0.89(0.00)	0.37	0.88(0.00) -0.13(0.53)	0.45	0.24(0.05) -0.09(0.21)	0.69
R90 - dgerstar, R90 - rstar2sided	1.12(0.00) -1.07(0.00)	0.48	0.84(0.00) -0.12(0.52)	0.45	0.24(0.04) -0.10(0.16)	0.69

Table 6: In-Sample estimation, eight quarters ahead autoregressive model

	Total inflation		Core inflation	
Variables	Coefficients and P-values	$\overline{R}^2$	Coefficients and P-values	$\overline{R}^2$
R10 - R90	-0.37(0.02)	0.46	0.04(0.60)	0.71
R90 - dgerstar	0.49(0.00)	0.50	0.01(0.84)	0.71
R90 - rstar1sided	0.17(0.04)	0.40	-0.05(0.34)	0.72
R90 - rstar2sided	0.12(0.19)	0.37	-0.06(0.17)	0.73
R10 - r90, (R10 - R90)2	-0.36(0.03) -0.01(0.91)	0.45	0.09(0.04) -0.07(0.00)	0.76
R90 - dgerstar, (R90 - dgerstar)2	0.80(0.05) -0.08(0.34)	0.50	0.04(0.82) -0.01(0.87)	0.71
R90 - rstar1sided, (R90 -rstar1sided)2	0.23(0.26) -0.01(0.77)	0.39	0.13(0.18) -0.03(0.02)	0.74
R90 - rstar2sided, (R90 - rstar2sided)2	0.17(0.47) -0.01(0.80)	0.36	0.14(0.19) -0.04(0.02)	0.75
R90 - dgerstar, R90 - rstar1sided	0.83(0.00) -0.23(0.06)	0.52	0.20(0.07) -0.14(0.05)	0.73
R90 - dgerstar, R90 - rstar2sided	0.86(0.00) -0.27(0.01)	0.54	0.20(0.06) -0.15(0.02)	0.75

Table 7: RMSE, non-autoregressive model

Variables	GDP	Total Inflation		Core Inflation	
	Four quarters	Four quarters	Eight quarters	Four quarters	Eight quarters
R10 - R90	1.99	1.28	1.01	1.13	1.33
R90 - dgerstar	2.03	1.64	1.44	1.19	1.22
R90 - rstar1sided	2.30	1.55	1.31	1.17	1.29
R90 - rstar2sided	2.09	1.48	1.39	1.24	1.39
R10 - r90, (R10 - R90)2	1.85	1.41	1.15	1.20	1.32
R90 - dgerstar, (R90 - dgerstar)2	2.07	1.68	1.49	1.22	1.24
R90 - rstar1sided, (R90 -rstar1sided)2	1.53	1.44	1.32	1.30	1.24
R90 - rstar2sided, (R90 - rstar2sided)2	1.62	1.58	1.48	1.35	1.41
R90 - dgerstar, R90 - rstar1sided	2.21	1.67	1.43	1.29	1.40
R90 - dgerstar, R90 - rstar2sided	1.80	1.64	1.51	1.39	1.53

Table 8: RMSE, autoregressive model

Variables	GDP	Total Inflation		Core Inflation	
	Four quarters	Four quarters	Eight quarters	Four quarters	Eight quarters
R10 - R90	2.02	1.23	1.19	0.61	0.77
R90 - dgerstar	2.13	1.21	1.18	0.56	0.53
R90 - rstar1sided	2.33	1.33	1.23	0.56	0.57
R90 - rstar2sided	2.08	1.20	1.26	0.58	0.61
R10 - r90, (R10 - R90)2	1.91	1.35	1.32	0.63	0.74
R90 - dgerstar, (R90 - dgerstar)2	1.96	1.24	1.32	0.60	0.61
R90 - rstar1sided, (R90 -rstar1sided)2	1.74	1.27	1.25	0.78	0.67
R90 - rstar2sided, (R90 - rstar2sided)2	1.68	1.19	1.24	0.63	0.55
R90 - dgerstar, R90 - rstar1sided	2.31	1.26	1.15	0.61	0.64
R90 - dgerstar, R90 - rstar2sided	1.86	1.21	1.11	0.66	0.73

Figure 1. Neutral Interest Rates from Kalman Filter Estimation 1985:1 to 2004:1

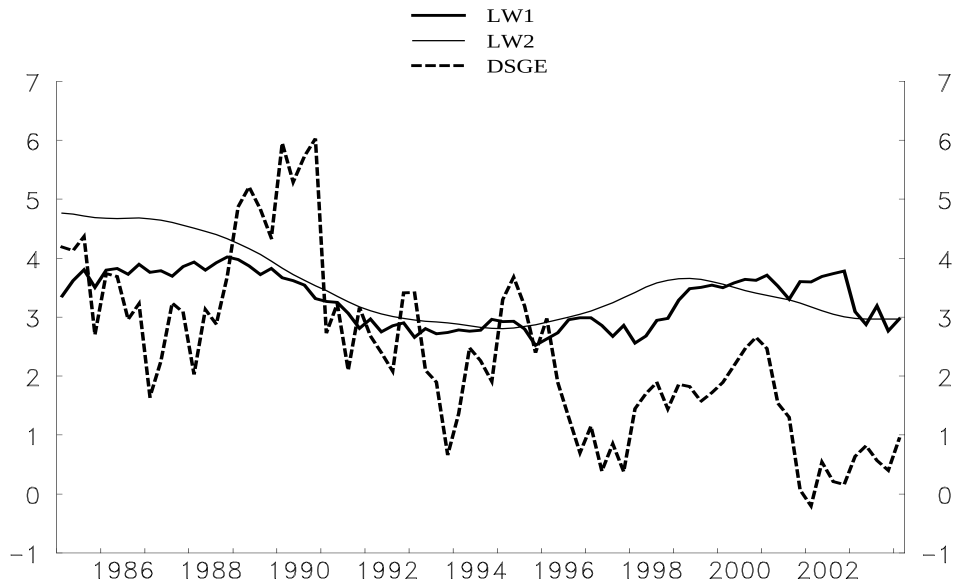
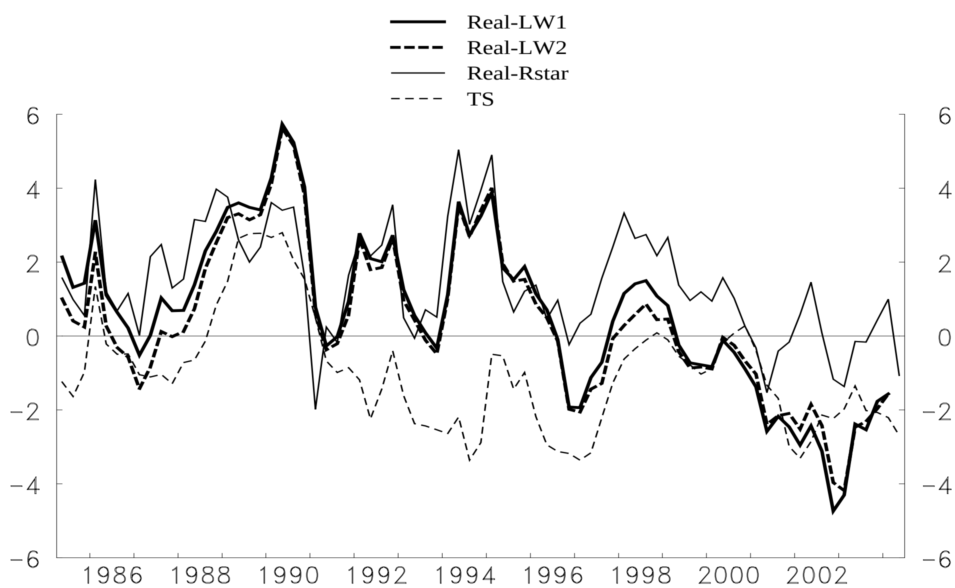


Figure 2. Real Interest Gap and the Term Spread 1985:2 to 2004:2



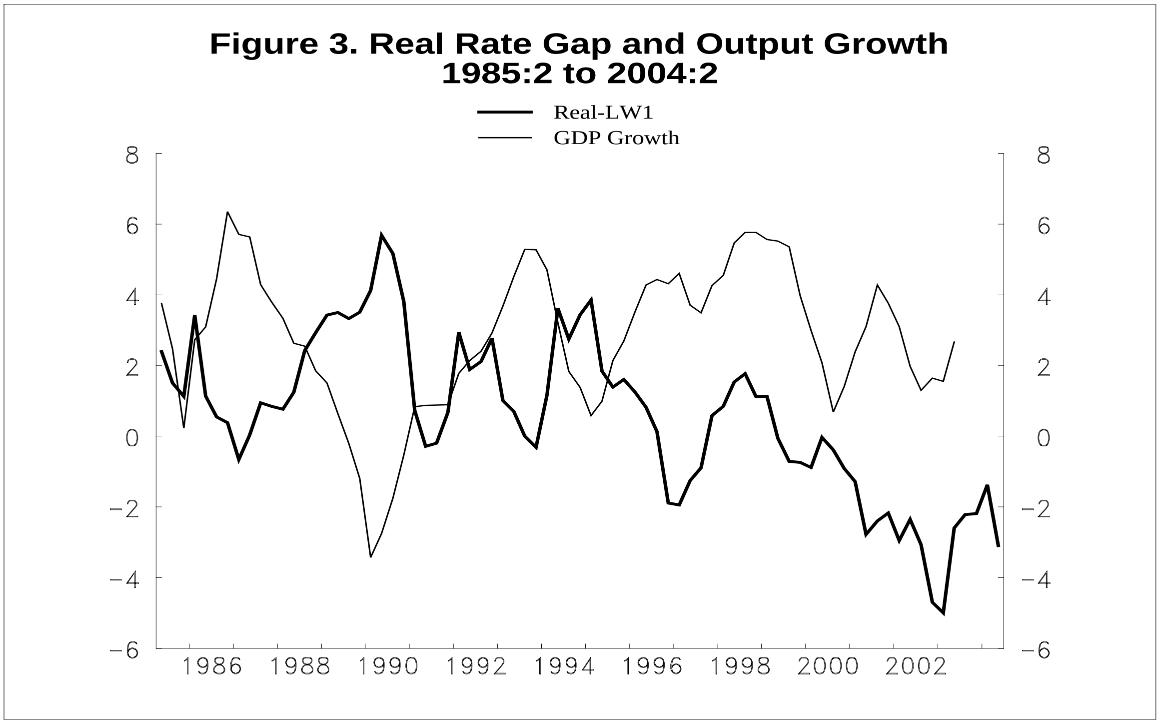


Figure 3. Real Rate Gap and Output Growth
1985:2 to 2004:2
Real-LW1
GDP Growth
8
6
4
2
0
−2
−4
−6
1986
1988
1990
1992
1994
1996
1998
2000
2002

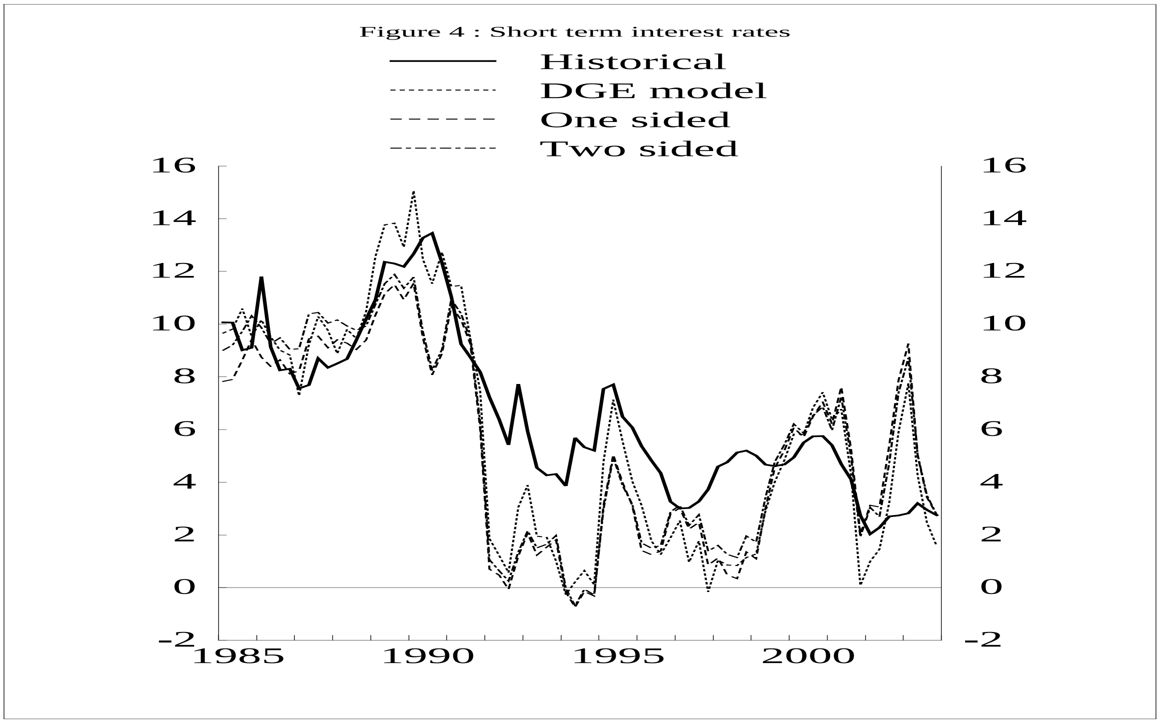


Figure 4 : Short term interest rates
Historical
DGE model
One sided
Two sided
16
14
12
10
8
6
4
2
0
-2
1985
1990
1995
2000

Figure 5 : One-sided estimate and confidence bands

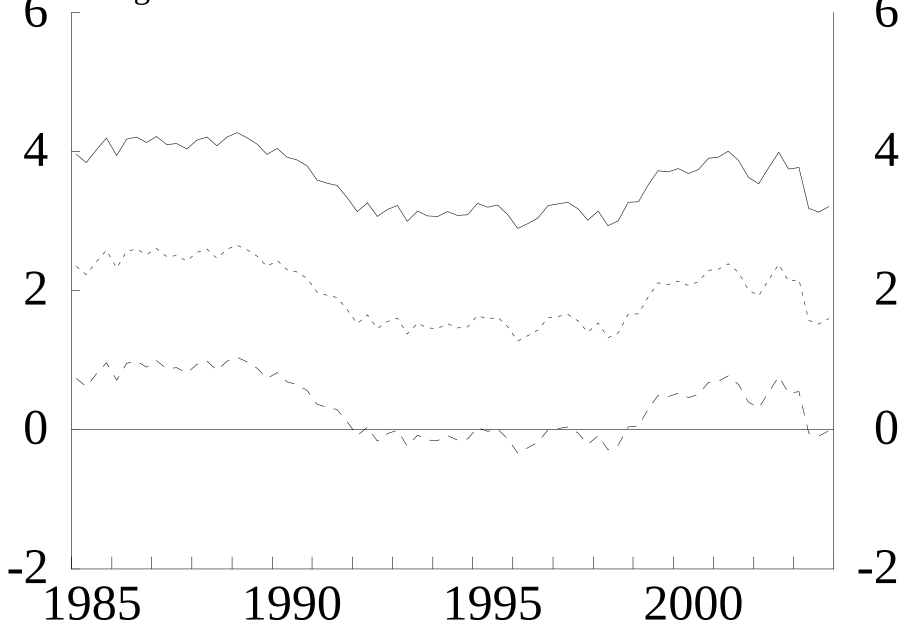


Figure 6 : Two-sided estimate and confidence bands

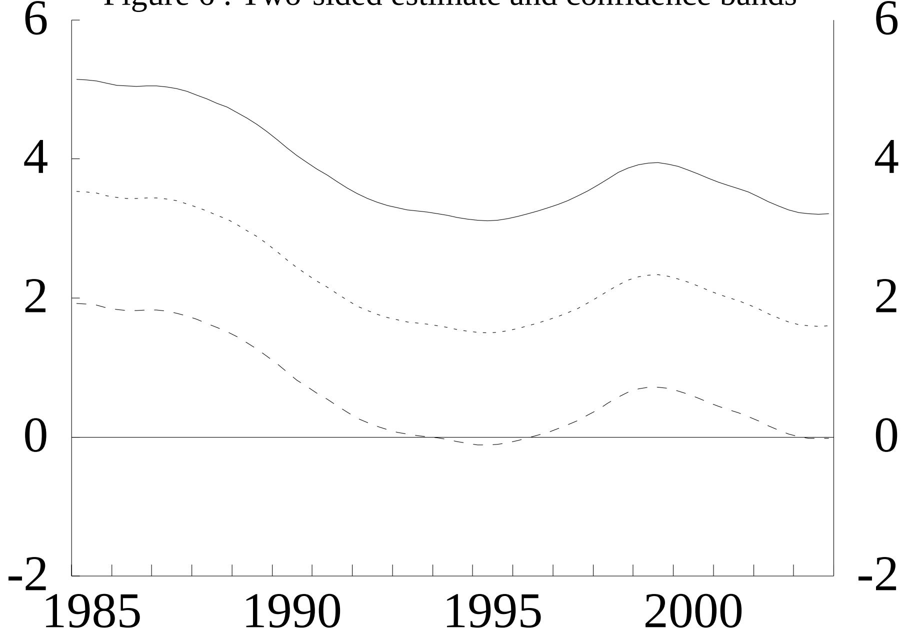


Figure 7 : Real time GDP

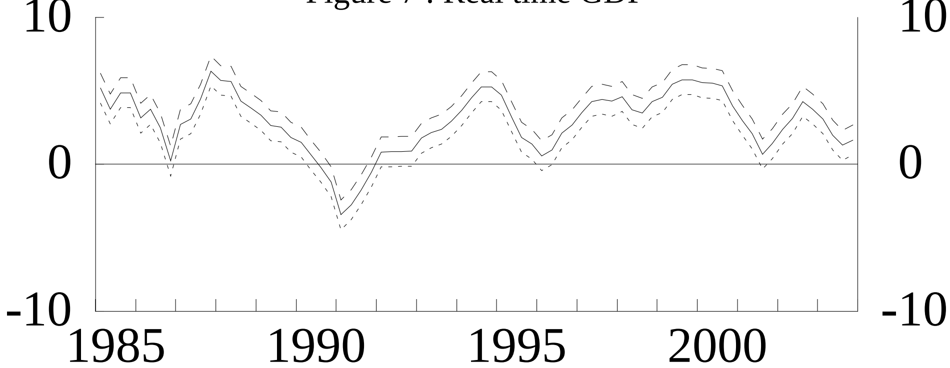


Figure 8 : Real neutral rate: one sided estimate

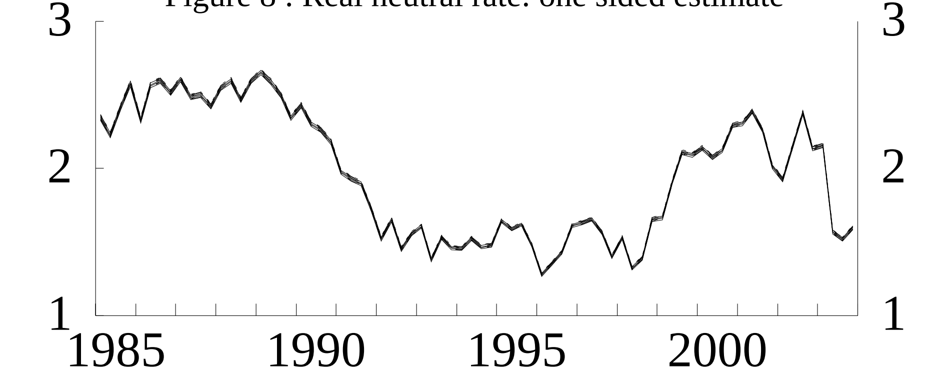


Figure 9 : Real neutral rate: two sided estimate

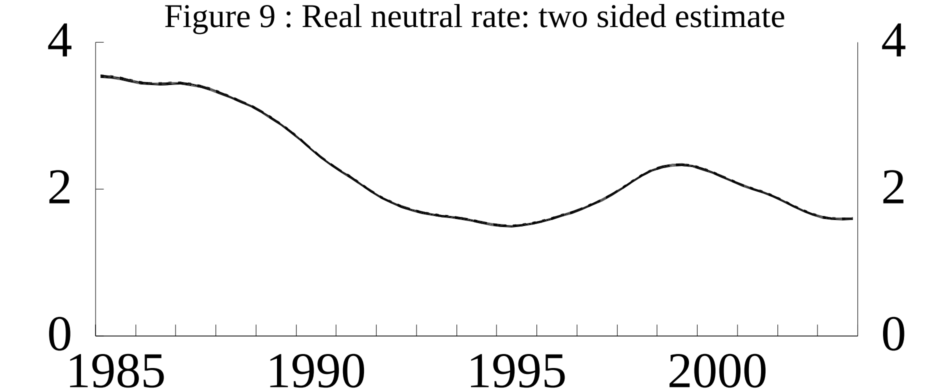


Figure 10: distribution of RMSE