Optimal Exchange Rate Policy in a Growing Semi-Open Economy [1]

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Abstract

In this paper, we consider an alternative perspective to China's exchange rate policy. When the private sector has no access to international capital markets, changes in central bank reserves fully reflect changes in a country's net saving. We analyze the optimal policy in such a context that we call a semi-open economy. We model the central bank as a Ramsey planner. We consider a model where limited financial development generates a large demand for saving instruments by the private sector. Our main result is that in a growth acceleration episode it is optimal to have an initial real depreciation of the currency combined with an accumulation of reserves, which is consistent with Chinese experience. This depreciation is followed by an appreciation in the long run. We also show that the optimal exchange rate path is similar to the one that woud result in an economy with full capital mobility and no central bank intervention.

1 Introduction

In recent years we have seen a heated debate on Chinese exchange rate policy and the enormous accumulation of international reserves by its central bank. While the increase in reserves has been considered as a major contributor to global imbalances, the renminbi (RBM) has typically been viewed as undervalued.[1] For example, Frankel (2010) clearly states "An appreciation would improve economic welfare". However, these views are not universally shared. For example, McKinnon (2010) gives two main arguments against more RMB flexibility. First, a flexible exchange rate is not desirable given the limited international use of the RMB. Second, an appreciation will not necessarily reduce the huge current account surplus, since it should reduce the difference between aggregate savings and aggregate investment.

This paper will focus on the second argument of McKinnon, namely the connection between the exchange rate level and net saving. We examine the optimal exchange rate policy in a dynamic intertemporal model that incorporates four basic features of the Chinese economy: i) limited capital mobility; ii) a net capital outflow taking the form of an accumulation of central bank international reserves; iii) underdeveloped financial markets; iv) a very high growth rate. In such a context the central bank is modeled as a Ramsey planner who can choose the optimal path of the exchange rate and of international reserves. Our main result is that in a growth acceleration episode it is optimal to have an initial real depreciation of the currency combined with an accumulation of reserves, which is consistent with Chinese experience. This depreciation is followed by an appreciation in the

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[1]For some recent contributions on this debate, see Cheung et al. (2011), Frankel (2010), or Goldstein and Lardy (2008).

long run. We also show that the optimal exchange rate path is similar to the one that would result in an economy with full capital mobility and no central bank intervention. The main reason for an optimal depreciation is financial underdevelopment implying a limited supply of financial assets. With a developed financial system, an initial appreciation would be optimal.

Studying the link between the real exchange rate and net saving naturally requires an intertemporal approach, in contrast to many analyses that examine the relationship between the exchange rate and the trade balance. The standard model analyzing this link is the representative individual infinite horizon model with traded and non-traded goods.[2] We deviate from this benchmark model to incorporate the four features mentioned above. The combination of the first two features gives us a "semi-open" economy, which is an economy where the private sector does not have access to the international capital market, but the central bank does. It has been argued that in this context the central bank can serve as intermediary between the private sector and the international capital market to allow for intertemporal trade (e.g., see Song et al., 2011). In a recent paper, Jeanne (2012) considers a semi-open economy with traded and non-traded goods and shows how exogenous changes in international reserves alter intertemporal consumption choices, as well as the real exchange rate.

The combination of the two other features, low financial development and high growth rates, potentially leads to an excess demand for saving instruments.[3] To model this saving need, we follow Woodford (1990) and introduce heterogeneous

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[2]See Obstfeld and Rogoff, 1996, ch. 4. For example, Obstfeld and Rogoff (2000, 2007) use this standard framework to analyze US net saving and the dollar exchange rate.

[3]Several other factors explain high saving in China (e.g. see Yang et al, 2011). Adding these factors would not change the main results of our analysis.

households who could be either borrowers or lenders. With strong credit frictions, lenders may not be willing to lend all their saving to borrowers. Consequently, there is a lack of financial assets for lenders since they do not have access to international capital markets.[4] However, the government or the central bank can provide domestic assets to accommodate the saving need. A natural way of changing the amount of domestic assets is for the central bank to serve as intermediary between the international capital market and domestic savers. Thus, an accumulation of international reserves at the central bank can be translated into an increase in the supply of domestic assets and an increase in private saving.[5] This will also affect the real exchange rate. A higher saving rate reduces demand and pressure on domestic prices, which implies a real depreciation. Therefore the optimal exchange rate policy is directly tied to asset provision and reserve policy.

As in several recent papers,[6] one feature of our analysis is the interaction between real exchange rate movements and a credit constraint. It is well know that this feature creates pecuniary externalities and therefore a role for policy intervention. It turns out, however, that this effect plays little role in our context and is dominated by several other effects. On the other hand there is no other externality from exchange rate movements. In contrast, Korinek and Serven (2011) and Benigno and Fornaro (2012) assume learning by doing in the export sector, which gives an incentive for currency depreciation and reserve accumulation.

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[4]This implies that a capital account liberalization would lead to a net private capital outflow. Several papers in the literature predict such an outcome using totally different perspectives. E.g. see He et al. (2012a).

[5]Since 2000, the Chinese central bank has increased its liabilities with the domestic banking sector at about the same rate as international reserves. These liabilities mainly take the form of central bank bonds and commercial banks reserves.

[6]E.g., Bianchi (2011), Korinek (2011), Benigno et al. (2012). Cespedes et al. (2012) examine central bank intervention with such an externality in the context of capital inflows.

To analyze the optimal exchange rate policy in the context of a semi-open economy, we take a dynamic optimal taxation approach, by modeling the authorities as a Ramsey planner. Although this approach has not been used for exchange rate policy (and even less in the Chinese context), there is growing interest in using these tools in international macroeconomics.[7] In a growth acceleration episode, we find that it is optimal to supply more domestic assets financed by international reserves. Therefore it is also optimal to let the currency depreciate. We also find that the optimal depreciation with capital controls is similar to the depreciation that would occur endogenously in an open economy. The need for reserve accumulation and currency depreciation will be stronger when the lack of saving instruments is acute, *i.e.* with low financial development.

This structural approach to optimal exchange rate policy therefore provides an alternative to the mainstream policy view that focuses on the link between the exchange rate and the trade balance. There are at least three advantages to this approach. First, exchange rate policy can be reconciled with saving and investment behavior. Second, an explicit dynamic welfare analysis can be conducted. Third, the role of structural factors can be examined.

In the following section, we lay out the model. The structure of the semi-open economy is similar to Bacchetta et al. (2012), but we consider traded and non-traded goods to determine real exchange rate movements. Section 3 describes the model equilibrium. Section 4 describes the Ramsey problem and derives several analytical results about the optimal policy. Section 5 presents numerical simulations and Section 6 concludes.

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[7]Fahri et al. (2012) show that a simple combination of taxes can replicate nominal exchange rate policy, but they do not consider a Ramsey planner.

2 Model

The economy is inhabited by infinitely-lived households who receive endowments in traded and non-traded goods and consume both goods. The relative price of non-traded goods in terms of traded goods, p_t, is the real exchange rate.[8] Following Woodford (1990, section I), endowments alternate between low and high levels and there are two groups of mass one of households. This structure implies that in a given period half of households have a high endowment and typically would like to save, while the other half have a low endowment and would like to borrow.[9] Households trade one-period local assets. Without loss of generality, these assets are denominated in the traded good.[10] There is a gross interest rate r_t (measured in traded goods) on lending and borrowing.

We assume that households do not have access to international capital markets. Therefore, high-endowment households can save either by lending to low-endowment households or by holding domestic assets.[11] However, high-endowment households may be reluctant to lend to other households due to credit market frictions and may thus be looking for other saving instruments.

In addition to households there is a Ramsey planner, that we call a central bank, who can issue local assets and hold international reserves, thereby affecting

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[8]In general there can be differences between the relative price of traded and non-traded goods and the commonly measured real exchange rate. We will abstract from these differences. He et al. (2012b) show that in the case of China the relative price of traded and non-traded goods shows a stronger appreciation in recent years than standard real exchange rates measures.

[9]There are four basic differences with Woodford (1990): i) consumers may be able to borrow; ii) there is a Ramsey planner; iii) there is no capital stock; iv) there are traded and non-traded goods.

[10]In the absence of uncertainty, the denomination of assets has no consequence on equilibrium allocations.

[11]In reality, the lending between high and low endowment households goes through the banking sector, with bank deposits and bank loans. Modeling financial intermediaries would not affect our analysis.

the real exchange rate. When credit constraints are tight, the opportunities to save for high-endowment households are limited. In this case the provision of local assets by the central bank may be desirable.

2.1 Households

At time t, a first group of households receives an endowment $Y_t = Y_t^T + p_t Y_t^N$, where the superscripts T and N stand for traded and non-traded. The second group receives aY_t, with $0 \leq a < 1$. At $t+1$, the first group receives aY_{t+1} while the second receives Y_{t+1}, and so on. We refer to the group with Y as cash-rich households, or savers, and the group with aY as cash-poor households, or borrowers. Each household alternates between a cash-rich and a cash-poor state, and each period there is an equally-sized population of rich and poor. Cash-rich households will hold assets A, while cash-poor households borrow L. Households also receive a profit from the central bank. These profits are distributed equally between the two groups so that each household receives $\pi_t/2$ in traded goods at period t. Profits can be negative, in which case households pay a lump-sum tax.

Households maximize:

$$\sum_{s=0}^{\infty} \beta^s u(c_s^T, c_s^N) \tag{1}$$

We will focus on separable iso-elastic utility functions $u(c_s^T, c_s^N) = v(c_s^T) + \kappa v(c_s^N)$ with

$$v(c) = \frac{c^{1-\sigma}}{1-\sigma} \qquad\qquad \text{for } \sigma \neq 1$$

$$v(c) = \ln c \qquad\qquad \text{for } \sigma = 1.$$

We denote consumption of traded (non-traded) goods during the cash-rich period as c^{AT} (c^{AN}). Consumption of traded (non-traded) goods during the cash-poor period is denoted c^{LT} (c^{LN}). Consider a household that is cash-rich at time t and cash-poor at date $t+1$. His budget constraints at t and $t+1$ are:

$$Y_t - r_t L_t + \pi_t/2 = c_t^{AT} + p_t c_t^{AN} + A_{t+1}, \tag{2}$$

$$aY_{t+1} + r_{t+1}A_{t+1} + \pi_{t+1}/2 = c_{t+1}^{LT} + p_{t+1}c_{t+1}^{LN} - L_{t+2}. \tag{3}$$

The household income at date t, which is composed of endowment Y_t minus debt repayments $r_t L_t$ plus central bank profits, is allocated to buying assets A_{t+1}, traded goods c_t^{AT}, and non-traded goods c_t^{AN}. In the following period, at $t+1$, its income is composed of the return on assets, $r_{t+1}A_{t+1}$, of aY_{t+1} and of central bank profits. This has to pay for consumption of traded and non-traded goods, c_{t+1}^{LT} and c_{t+1}^{LN}. Typically the cash-poor household will borrow, so that at the optimum $L_{t+2} \geq 0$.

The cash-poor household might face a credit constraint when borrowing at date $t+1$. Due to standard moral hazard arguments, a fraction $0 \leq \phi < 1$ of the total endowment is used as collateral for bond repayment:

$$r_{t+2}L_{t+2} \leq \phi Y_{t+2} \tag{4}$$

The multiplier associated with this constraint is denoted $v'(c_{t+2}^{AT})\lambda_{t+2}$.

Cash-rich households at time t satisfy the following Euler equation:

$$v'(c_t^{AT}) = \beta r_{t+1} v'(c_{t+1}^{LT}) \tag{5}$$

Similarly, poor households at date t satisfy the following Euler equation:

$$v'(c_t^{LT}) = \beta r_{t+1} v'(c_{t+1}^{AT}) (1 + \lambda_{t+1}) \tag{6}$$

The intertemporal choice of a cash-poor household is distorted when the credit constraint is binding, because $\lambda_{t+1} > 0$. The following slackness condition has also to be satisfied:

$$(\phi Y_{t+1} - r_{t+1} L_{t+1}) \lambda_{t+1} = 0 \tag{7}$$

2.2 The Real Exchange Rate

The first order conditions give:

$$p_t = \kappa \frac{v'(c_t^{LN})}{v'(c_t^{LT})} = \kappa \frac{v'(c_t^{AN})}{v'(c_t^{AT})} \tag{8}$$

In equilibrium total non-traded consumption is equal to total non-traded endowment:

$$c_t^{AN} + c_t^{LN} = (1 + a) Y_t^N \tag{9}$$

In this case, the first-order conditions imply:

$$p_t = \kappa \left(\frac{c_t^{AT} + c_t^{LT}}{(1 + a) Y_t^N} \right)^\sigma \tag{10}$$

Since this is an endowment economy, the real exchange rates simply depends on the ratio between traded consumption and non-traded output. The evolution of traded good consumption is obviously affected by the presence of credit constraints. Consider for example an increase in the growth rate of all endowments.

As we shall see, the credit constraint then implies higher saving so that $c_t^{AT} + c_t^{LT}$ increases initially less than the endowment. This implies a decline in p_t and thus a depreciation.

The depreciation in a period of strong growth is thus associated with an increase in saving. How is this possible in the aggregate? In an open economy households would buy foreign assets. In a semi-economy, this is possible if the central bank issues local assets, financed by the accumulation of reserves. Thus, as shown by Jeanne (2012), the accumulation of reserves is directly related to saving and to the exchange rate. In this paper we will determine the optimal exchange rate/reserves policy.

2.3   Central Bank Policy

The central bank issues domestic assets B_{t+1} at time t paying an interest rate r_{t+1}. It has access to foreign reserves B_{t+1}^* (denominated in traded goods) that yield the world interest rate r^*. We assume that $r^* = 1/\beta$. Private agents cannot buy external bonds directly, so the domestic interest rate is determined in the domestic bond market. Equilibrium in this market is:

$$B_{t+1} = A_{t+1} - L_{t+1} \tag{11}$$

In the presence of capital controls, only the central bank has access to external assets, so it has a monopoly over the supply of bonds to domestic agents. It can therefore manipulate the domestic interest rate r_{t+1} by appropriately setting the supply of bonds B. The possibility of accumulating reserves B^* enables the central bank to change the domestic supply of bonds by simply expanding its

balance sheet. The central bank can then match the desired domestic saving by accumulating reserves.

When the central bank policy creates a wedge between r_{t+1} and r^*, this generates revenues or losses. We assume that the central bank transfers directly its profits π_t to households.[12] The central bank budget constraint is:

$$B^*_{t+1} + r_t B_t + \pi_t = r^* B^*_t + B_{t+1}. \tag{12}$$

We impose the usual no-ponzi condition to the central bank net asset position:

$$\lim_{T \to \infty} \frac{B^*_T - B_T}{(r^*)^T} = 0. \tag{13}$$

In general, profits $\{\pi_t\}_{t \geq 0}$ have to satisfy the sequence of budget constraints (12) and the no-ponzi condition (13) given the policy $\{B_{t+1}, B^*_{t+1}\}_{t \geq 0}$. In the following, we focus on the benchmark case where the central bank transfers its revenue or losses to households on a period-by-period basis:

$$\pi_t = (r^* - 1)B^*_t - (r_t - 1)B_t. \tag{14}$$

With this assumption, a change in international reserves has to be matched by an increase in the supply of bonds: $B^*_{t+1} - B^*_t = B_{t+1} - B_t$. Assuming that $B^*_0 = B_0$, we have:

$$B^*_t = B_t. \tag{15}$$

Remark that the closed economy and the open economy are special cases nested

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[12]In practice, central banks usually transer they profits to the government, which relaxes the government budget constraint.

in our semi-open economy framework. The central bank can always choose to "replicate" the open economy by supplying the domestic market with bonds at the world interest rate $r_{t+1} = r^*$. It can also mimic the closed economy by not buying reserves: $B_{t+1}^* = 0$.

As a Ramsey planner, the central bank will choose a policy $\{B_{t+1}, B_{t+1}^*\}_{t \geq 0}$ to maximize its social objective:

$$\sum_{s=0}^{\infty} \beta^s \left[u(c_s^{AT}, c_s^{AN}) + u(c_s^{LT}, c_s^{LN})) \right] \tag{16}$$

We will then analyze the optimal exchange rate policy in this context.

3 Competitive equilibrium

In this section, we examine the properties of a competitive equilibrium for a given policy. First, we describe how the reserve policy is equivalent to an exchange rate policy and how it affects the bond market. Then, we analyze the steady state and determine the conditions under which the economy is constrained.

We define a competitive equilibrium as follows:

Definition 1 (Competitive equilibrium) *Given endowment streams $\{Y_t^T, Y_t^N\}_{t \geq 0}$ and initial conditions $r_0, A_0, L_0, B_0, B_0^*$ with $B_0 = A_0 - L_0$, a competitive equilibrium is a sequence of prices $\{p_t, r_{t+1}\}_{t \geq 0}$ and Lagrange multipliers $\{\lambda_{t+1}\}_{t \geq 0}$, an allocation $\{A_{t+1}, L_{t+1}, c_t^{AT}, c_t^{LT}, c_t^{AN}, c_t^{LN}\}_{t \geq 0}$, and a policy $\{\pi_t, B_{t+1}, B_{t+1}^*\}_{t \geq 0}$ such that: (i) given the price system and the policy, the allocation and the Lagrange multipliers solve the households' problems (equations (2)–(7) are satisfied); (ii) given the allocation and the price system, the policy*

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satisfies the sequence of central bank budget constraints (12) and the no-ponzi condition (13); (iii) the markets for non-traded goods (9) and domestic bonds (11) clear.

As explained earlier, we will restrict the analysis to the subset of policies defined by the profit distribution rule (14) and assume that $B_0^* = B_0$ so that the holding of reserves equals the supply of bonds by the central bank (15).

3.1 Central Bank Policy, the Real Exchange Rate, and the Real Interest Rate

With separable iso-elastic utility, intratemporal optimization by households implies that the real exchange rate depends on the aggregate consumption of traded goods as shown by equation (10). Using the budget constraints (2), (3) and (12) together with the market-clearing conditions (9) and (11), we can derive a current account identity:

$$B_{t+1}^* - B_t^* = (1+a)Y_t^T + (r^* - 1)B_t^* - (c_t^{AT} + c_t^{LT}). \tag{17}$$

Substituting equation (17) into (10), we clearly see how choosing the increase in reserves $B_{t+1}^* - B_t^*$ is equivalent to setting the real exchange rate p_t:

$$p_t = \frac{\kappa}{(1+a)Y_t^N} \left[(1+a)Y_t^T + (r^* - 1)B_t^* - (B_{t+1}^* - B_t^*) \right]^\sigma. \tag{18}$$

By buying more reserves, and issuing the corresponding amount of domestic bonds, the central bank can depreciate the real exchange rate, as explained in Jeanne (2012): in the semi-open economy, reserve policy and exchange rate policy

are equivalent.

While accumulating more reserves during the transition, *i.e.* choosing a higher flow $B_{t+1}^* - B_t^*$, depreciates the real exchange rate, a larger *stock* of reserves in the steady state appreciates the real exchange rate if $r^* > 1$, as it makes domestic agents richer and increase their demand for non-traded goods. In the steady state, equation (18) can indeed be rewritten as

$$p = \kappa \left[\frac{Y^T}{Y^N} + (r^* - 1) \frac{B_t^*}{(1+a)Y^N} \right]^\sigma .$$

The exchange rate policy also has an effect on the domestic bond market and the domestic interest rate. Since the stock of reserves is equal to the supply of domestic bonds by the central bank, depreciating the exchange rate requires increasing the supply of bonds. This leads to a higher domestic interest rate. Such a policy might be desirable when borrowing constraints are binding.

To see this, consider the demand for assets by savers in the case of log utility ($\sigma = 1$) where we can get closed-form solutions:[13]

$$A_{t+1} = \frac{1}{1+\beta} \left(\beta(Y_t - r_t L_t + \pi_t/2) - \frac{aY_{t+1} + \pi_{t+1}/2}{r_{t+1}} - \frac{L_{t+2}}{r_{t+1}} \right) \qquad (19)$$

The effect of a binding borrowing constraint is to decrease future borrowing L_{t+2}, which leads to a larger demand for saving instruments A_{t+1}. At the same time, a binding borrowing constraint also decrease current borrowing by cash-poor households L_{t+1}, as implied by (7) when it holds as an equality. Absent any policy intervention, the excess demand for and the constrained supply of bonds by the

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[13]This equation follows from the Euler equation (5) and the budget constraints (2) and (3).

private sector would lead to an abnormally low interest rate r_{t+1} to clear the market, compared with a frictionless economy. By providing more bonds to the domestic market, a policy of real exchange rate depreciation can alleviate the limited supply of bonds by cash-poor households and accommodate the need for saving by cash-rich households.

3.2 Symmetric Steady States

How central bank policy can alleviate borrowing constraints by providing domestic bonds can be analyzed precisely in deterministic symmetric steady states, defined as follows.

Definition 2 (Symmetric Steady State) *Consider a constant endowment stream $(Y_t^T, Y_t^N) = (Y^T, Y^N)$ for $t \geq 0$. A symmetric steady state is a constant price vector (p, r), Lagrange multiplier λ, allocation $(A, L, c^{AT}, c^{LT}, c^{AN}, c^{LN})$, and policy (π, B, B^*) that form a competitive equilibrium associated to the endowment stream (Y^T, Y^N) and the initial conditions r, A, L, B, B^*.*

In a symmetric steady state, endowments and consumptions of a given individual can still fluctuate through time; but their distributions across agents are stationary. Such a steady state is symmetric in the sense that all individuals have the same state-contingent consumption and wealth.

Define the following reduced parameter $\bar{b}$:

$$\bar{b} = \frac{\beta(1 + \kappa)\left(\frac{1-a}{1+\beta} - 2\phi\right)}{1 - \frac{\kappa(1-\beta)}{1+a}\left(\frac{1-a}{1+\beta} - 2\phi\right)}.$$

The denominator of $\bar{b}$ is strictly positive when $\kappa < \frac{1+\beta}{1-\beta}\frac{1-a}{1+a}$, a weak condition which we assume throughout.[14]

The following proposition shows that the steady states of the model depend on how the amount of bonds B/Y^T compares to $\bar{b}$.

Proposition 1 *Assume the profit distribution* (14) *and log utility. For all* $(Y^T, Y^N, B^*) \in \Re^{+*2} \times \Re^+$, *there is a unique symmetric steady state.*

If $\frac{B^*}{Y^T} < \bar{b}$, the credit constraint is binding, the interest rate $r < r^*$ increases with $\frac{B^*}{Y^T}$ and the ratio of relative traded consumption is given by $\frac{c^{LT}}{c^{AT}} = r/r^* < 1$.

If on the contrary $\frac{B^*}{Y^T} \geq \bar{b}$, the credit constraint does not bind and $r = r^*$.

Proof. See Appendix A. ■

The proposition shows how the accumulation of reserves, or equivalently the issuance of domestic bonds, determines the extent to which households can smooth consumption despite the borrowing constraint. A higher level of reserves B^* and domestic bonds B means that cash-rich households can save more and receive a larger return on their saving, resulting in smaller fluctuations of tradable consumption through time. When the supply of bonds is large enough, cash-rich households can accumulate enough assets to completely overcome their borrowing constraint and perfectly smooth consumption.

A direct corollary of Proposition 1 is that the borrowing constraint never binds in a steady state of the open economy and that the net foreign asset position of an open economy, B^*, is necessarily larger than $\bar{b}Y^T$ in a steady state. For stringent enough borrowing constraints (*i.e.* low enough ϕ), $\bar{b}$ is positive, and the open economy has positive net foreign assets in the steady state.

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[14]For example, with log-utility and $\beta = 0.95$, this condition holds as long as tradable consumption represents at least 2.5% of total consumption.

4 Optimal Exchange Rate Policy

4.1 The Ramsey Problem

To analyze optimal policy we now turn to the optimization problem of the Ramsey planner. We consider the log utility case. Without loss of generality, we assume zero initial net assets ($B_0^* - B_0 = 0$). The planner maximizes its objective (16) subject to the household budget constraints, their first order conditions, the borrowing constraint, the complementary slackness condition, the market-clearing conditions for bonds, and the resource constraint for both non-tradable goods (given by the market-clearing condition (9)) and tradable goods (given by the current account identity (17)).[15] Using the optimality conditions, the value of non-tradable consumption in terms of tradables is suppressed from the Ramsey program, namely $p_t c_t^{AN} = \kappa c_t^{AT}$ and $p_t c_t^{LN} = \kappa c_t^{LT}$.

Maximization is then carried out with respect to $\{L_{t+1}, A_{t+1}, c_t^{AT}, c_t^{LT}, r_{t+1}, p_t, \lambda_{t+1}, \pi_t, B_t^*\}_{t \geq 0}$. The Lagrangian of the Ramsey

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problem in the semi-open economy is then defined as follows:

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\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & (1+\kappa)\ln(c_t^{AT}) + (1+\kappa)\ln(c_t^{LT}) - 2\kappa\ln p_t \\
& +\gamma_t^A \left[Y_t^T + p_t Y_t^N - (1+\kappa)c_t^{AT} + \pi_t/2 - A_{t+1} - r_t L_t \right] \\
& +\gamma_t^L \left[a(Y_t^T + p_t Y_t^N) + r_t A_t + L_{t+1} - (1+\kappa)c_t^{LT} + \pi_t/2 \right] \\
& +\gamma_t^G \left[r^* B_t^* - (B_{t+1}^*) + (1+a)Y_t^T - c_t^{AT} - c_t^{LT} \right] \\
& +\gamma_t^B \left[B_{t+1}^* + L_{t+1} - A_{t+1} \right] \\
& +\gamma_t^N \left[(1+a)p_t Y_t^N - \kappa(c_t^{AT} + c_t^{LT}) \right] \\
& +\kappa_t^A \left[v'(c_t^{AT}) - \beta r_{t+1} v'(c_{t+1}^{LT}) \right] \\
& +\kappa_t^L \left[v'(c_t^{LT}) - \beta r_{t+1} v'(c_{t+1}^{AT})(1 + \lambda_{t+1}) \right] \\
& + \Gamma_t \left[\phi(Y_t^T + p_t Y_t^N) - r_t L_t \right] \\
& + \Delta_t \left[(\phi(Y_t^T + p_t Y_t^N) - r_t L_t)\lambda_t \right] \Bigg\}
\end{aligned}
$$

The planner takes as constraints both the borrowing constraint (which does not necessarily bind) and the complementary slackness condition, which both enter in the definition of the competitive equilibrium. It is useful to define $\Lambda_t = \Gamma_t + \lambda_t \Delta_t$.

While the full solution to this dynamic optimization has to be solved numerically, some interesting properties can be derived analytically. In particular, the steady state can be fully characterized. As regards transition dynamics, one can ask whether the planner wants to deviate from the closed economy regime characterized by $B^* = 0$ and a constant real exchange rate. One can also determine whether the planner wants to deviate from the open economy regime with $r = r^*$.

4.2 Optimal Accumulation of Reserves in the Steady State

To study the optimal accumulation of reserves, we focus on the first order condition with respect to B_{t+1}^*:

$$-(\gamma_t^G - \gamma_{t+1}^G) + \gamma_t^B = 0$$

Using the other FOCs of the planner's program, we can replace γ_t^B to get (see Appendix B for details):

$$-(\gamma_t^G - \gamma_{t+1}^G) + \beta r_{t+1} \frac{\Lambda_{t+1}}{2} = 0. \tag{20}$$

The first term reflects the usual motive of intertemporal smoothing. The Lagrange multiplier γ^G is the shadow cost of the resource constraint for tradable goods. When the tradable endowment is growing, this multiplier should decrease over time in the absence of policy intervention (*i.e.* in a closed economy with $B_t^* = 0$), making the first term negative. This first effect makes the planner want to borrow abroad and appreciate the real exchange rate. The second term captures the effect of the borrowing constraint. With a binding borrowing constraint, the planner wants to accumulate reserves and depreciate the exchange rate. The optimal policy balances those two effects.

When the borrowing constraint does not bind, both terms are equal to zero and borrowing abroad allows the planner to get a constant shadow cost γ^G and achieve perfect intertemporal smoothing. A binding borrowing constraint provides a motive to borrow less than in a frictionless economy, and to potentially accumulate reserves.

This can be seen clearly in a steady state. Then, the first term disappears and equation (20) simply becomes $\Lambda = 0$. The steady state optimal policy consists in completely relaxing borrowing constraints. Using Proposition 1, we can then characterize the optimal level of reserves in a steady state.

Proposition 2 *A steady state with optimal central bank policy has positive foreign reserves when $2\phi(1 + \beta) < 1 - a$.*

Proof. From equation (20) taken in the steady state, we have $\Lambda = 0$. Therefore, the borrowing constraint does not bind in the steady state. From Proposition 1, this implies $B^* \geq \bar{b}Y^T$. Given our assumption that $\kappa < \frac{1+\beta}{1-\beta}\frac{1-a}{1+a}$, the condition $2\phi(1 + \beta) < 1 - a$ implies $\bar{b} > 0$ and therefore $B^* > 0$. ∎

Remark that the steady state optimal policy consists in replicating an open economy with $r = r^*$.

4.3 Is the Closed Economy Optimal?

Consider now the case of transitory dynamics where endowments of both tradable and non-tradable goods grow at the rate g_t: $Y_{t+1}^T = (1 + g_{t+1})Y_t^T$ and $Y_{t+1}^N = (1 + g_{t+1})Y_t^N$. Assume that $a(1 + g_{t+1}) < 1$ so that endowments fluctuate still decline for cash-poor households.

To study the optimal reserve policy, we consider the closed economy and determine whether the planner wants to deviate from it. Denote by $\tilde{J}_{t+1}$ the left-hand side of (20) evaluated in the closed economy with $B_t = B_{t+1} = 0$. In general, any deviation of $\tilde{J}_{t+1}$ from zero means that the central bank can improve welfare by changing the level of reserves and the real exchange rate. When $\tilde{J}_{t+1}$ is posi-

tive, social welfare can be increased by buying reserves and depreciating the real exchange rate below its value in the closed economy.

The expression for $\tilde{J}_{t+1}$ can be solved explicitly in the case of a full borrowing constraint $\phi = 0$.[16] In Appendix C, we show that $\tilde{J}_{t+1}$ is then given by:

$$J_{t+1} = \frac{1+a}{2aY_{t+1}^T}\left(1 - \frac{\tilde{r}_{t+1}}{r^*}\right) \tag{21}$$

where $\tilde{r}_{t+1}$ is the closed-economy interest rate. The planner finds it socially optimal to accumulate reserves and depreciate the real exchange rate during the transitory dynamics, as long as the closed economy interest rate is strictly lower than the world interest rate.

It easy to see that $\tilde{r}_{t+1} < r^*$ under our assumption $a(1 + g_{t+1}) < 1$. Using the fact that $\pi_t = 0$ in the closed economy, the demand for bonds by savers (19) becomes

$$A_{t+1} = \frac{1}{1+\beta}\left(\beta Y_t - \frac{a(1+g_{t+1})Y_t}{\tilde{r}_{t+1}}\right)$$

Market clearing on the bond market implies $A_{t+1} = 0$ so that the closed-economy interest rate $\tilde{r}_{t+1}$ is given by $\beta\tilde{r}_{t+1} = a(1+g_{t+1})$. Since $r^* = 1/\beta$, we have $\tilde{r}_{t+1} < r^*$, so that reserve accumulation and currency depreciation are optimal when starting from the closed economy.

4.4 Is the Open Economy Optimal?

So far, we have shown that it is optimal to reproduce the open economy in the steady state and to accumulate reserves if one starts from a closed economy

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[16]From Proposition 2, we already know that it is optimal to accumulate reserves in the steady state when $\phi = 0$.

with tight borrowing constraints. An interesting question is whether the optimal reserve policy consists in simply replicating the open economy.

To answer this question, we evaluate the left-hand side of (20) at $r_{t+1} = r^*$. Let us denote this expression by J_{t+1}^*. Any deviation of J_{t+1}^* from zero means that the open economy is suboptimal and that the central bank can improve welfare by accumulating (or decumulating) reserves with respect to the open economy. When J_{t+1}^* is positive, social welfare can be increased by accumulating more reserves than the open economy. We obtain the following:

$$
J_{t+1}^* = \frac{1+\beta}{\beta c_t^{AT}} \left[\left(\sum_{i=1}^{\infty} \Lambda_{t+2i} \right) A_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+1+2i} \right) L_{t+1} - \frac{1}{2} \left(\sum_{i=1}^{\infty} \Lambda_{t+1+i} \right) (A_{t+1} - L_{t+1}) \right]
$$
$$
+ \kappa \left[-\frac{\phi}{1+a}(\Lambda_t - \Lambda_{t+1}) + \frac{2}{1+a} \left(\frac{1}{c_t^{LT} + c_t^{AT}} - \frac{1}{c_{t+1}^{LT} + c_{t+1}^{AT}} \right) - \frac{\Lambda_{t+1}}{2} \right] \quad (22)
$$

In the steady state, J_{t+1}^* converges to zero as Λ goes to zero and the consumption of tradables converges to its steady-state level. This confirms that an open economy in the steady state is at the Ramsey optimum. However, in the transition, the open economy could deviate from the optimum. To interpret condition (22), it is useful to remark that an increase (decrease) in reserves leads to a higher (lower) interest rate than in the open economy. Changes in the interest rates then affect the utility of both cash-rich and cash-poor agents.

The terms in the first square bracket (first line) of (22) arise whether there are nontradable goods in the economy or not (and are also present in Bacchetta et al., 2012). The first term corresponds to the net effect of the interest rate on savers and is positive, as they benefit from higher returns on saving, which alleviates their future constraints. The second term corresponds to the net effect on loan

issuers. This term is negative because a high interest rate hurts the borrowing households both through higher interest payment and through a more stringent credit constraint. The third term corresponds to the effect of central bank profits. Indeed, if $A > L$ and $r > r^*$, the interest payments on domestic debt are higher than the proceeds from external reserves, so that central bank profits are negative and households need to pay a lump sum tax to balance the budget.

In the contrast, the terms on the second line disappear if $\kappa = 0$, when agents do not value nontradables. They reflect the real exchange rate consequences of changing the level of reserves. Indeed, by accumulating reserves, the central bank depresses the real exchange rate. By doing so, the government makes the credit constraint more stringent, as long as creditors admit a share $\phi > 0$ of non-tradable goods as collateral. On the other hand, accumulating reserves increases the future consumption of tradable goods and stimulates the real exchange rate, alleviating future constraints. This explains the first term between brackets, which depends on $\Lambda_t - \Lambda_{t+1}$. The second term is positive if the domestic consumption of tradable is increasing, which happens only if households are constrained. In that case, the central bank wants to depreciate the price of nontradables in the short run in order to make households less willing to consume tradables, which alleviates the constraint.

To summarize, it is in general optimal to deviate from the open economy in the transition due to several effects. Where the deviations are large is a quantitative question that is examined in the next section.

5 Numerical Simulations of Optimal Policies

We examine the full solution to the Ramsey problem in two case. First, to illustrate the theoretical results in the previous section, we consider a constrained closed economy and determine its optimal path to its unconstrained steady state. Second, we analyze the optimal policy in a growth acceleration episode similar to the one experienced by the Chinese economy.

5.1 Real Exchange Rate Dynamics in Opening-up Economies

Consider a closed-economy characterized by strong borrowing constraints: $2\phi(1+\beta) < 1-a$. We know from Proposition 2 that in such a case, the steady state optimal policy consists in accumulating enough reserves to completely overcome the borrowing constraints. We illustrate this result numerically and examine the whole dynamics of the optimal policy. We simulate a baseline case, with $\kappa = 3$, $\phi = 0.1$, $a = 0$, $\beta = 1/1.05$, and no growth. For comparison purposes, we simulate the closed economy and the open economy, along with the optimal semi-open economy.

These dynamics are represented in Figure 1. Consider first the dynamics of the open economy, represented by the dashed line. In the long run, the economy converges to its unconstrained steady state with a higher level of foreign assets, which gives households the means to smooth their consumption of tradable goods. However, in the short-run, the economy does not have enough foreign assets yet and is constrained. As a result of the sharp increase in interest rate, cash-poor households are less able to borrow and have to decrease their consumption of trad-

ables. Anticipating this, cash-rich households cut on their tradable consumption in order to accumulate assets. Consequently, the price of nontradable goods depreciates on impact. As the economy accumulates foreign assets, the consumption of tradable goods increases and the price of nontradables appreciates. In the long run, the real exchange rate is slightly higher than in the closed economy steady state because the consumption of tradables is higher thanks to the positive foreign asset position.

Consider now the dynamics of the optimal semi-open economy, represented by the solid line. The economy converges to a similar unconstrained steady state with positive reserves. However, the interest rate does not jump immediately to the world level, but smoothly increases towards it. This gradual increase alleviates the negative effect of the interest rate on the borrowing capacities of cash-poor households, which stimulates their initial consumption. It also decreases debt repayments for cash-rich households and increases their consumption. As a result, the real exchange rate depreciates less on impact.

5.2 Real Exchange Rate Dynamics in Catching-up Economies

We now turn to the case of a growing economy. We assume that the economy experiences persistent growth but converges to a stationary steady state: $g_{t+1} = \mu g_t$. This corresponds to a catching-up economy. Importantly, tradable and nontradable endowments grow at the same rate. We consider the same baseline case as before, with $\kappa = 3$, $\phi = 0.1$ and $a = 0$, and choose $\mu = 0.9$. We start from a symmetric steady state where agents are marginally unconstrained.

At $t = 0$, the economy is hit by a positive growth shock $g_0 = 10\%$.

The optimal semi-open economy dynamics are presented in Figure 2. As agents are credit constrained in their cash-poor period, they accumulate assets A in their cash-rich period. This accumulation is made possible by an increase in B and thus in net foreign assets B^*. Notice that the increase in B^* is so strong that the domestic interest rate r_t rises above r^*. As the consumption of tradable goods is initially depressed relatively to the consumption of nontradables because of this accumulation of foreign assets, the real exchange rate initially depreciates. However, as the accumulation of foreign assets increases the tradable revenues of the economy relative to nontradables, the real exchange rate starts appreciating after a few periods. Our model therefore features an appreciating currency in catching-up economies, similar to a Balassa-Samuelson effect. But contrary to the Balassa-Samuelson effect, this appreciation is not generated by TFP catch-up in the tradable sector (we assume the same growth rate in both sectors) but by credit constraints. The interest rate is temporarily higher than in the rest of the world, which implies that the government accumulates more assets than in the open economy. This helps cash-poor households as it increases the return on their savings.

In order to assess the role of policy, we compare the dynamics of the real exchange rate in the optimal semi-open economy and in the open economy, both in the baseline calibration. The results are represented in the upper-left panel of Figure 3. The real exchange rate has a similar behavior in the open and semi-open economy. This suggests that the initial depreciation followed by an appreciation are natural outcome of a growth acceleration in a credit-constrained economy and would occur without policy intervention. The only difference is that, in the

optimal semi-open economy, the real exchange rate is slightly less depreciated as the government is able to somewhat alleviate the credit constraints. But this is the case only after a few periods, as in the beginning the government accumulates more foreign assets than in the open economy, which depresses the consumption of tradables and depreciates the real exchange rate.

As sensitivity checks we consider the cases with a less persistent growth episode, $\mu = 0.75$, and a stronger preference for non-tradables, $\kappa = 4$. The case with a smaller μ is represented in the middle-left panel of Figure 3. The dynamics of p is similar to the baseline, except that the initial depreciation is smaller and shorter. Indeed, with less persistent growth, the constraints are less binding, which mitigates the initial depreciation. The case with a larger κ is represented in the lower-left panel. Qualitatively, the dynamics of the real exchange rate is similar to the baseline case except that the extent of the initial real depreciation is larger, as agents value more nontradable

To further asses the role of credit constraints, we compare these dynamics to those obtained when the agents can pledge a larger share of their income as collateral. We consider the case where $\phi = 0.3$, which is represented by the upper-right panel of Figure 3. Here the constraints are sufficiently lax so that the economy borrows as a whole. The dynamics of the real exchange rate are now reversed: the country experiences first an appreciation and then a depreciation. Indeed, agents are now able to better smooth their consumption of tradables, which is impossible for nontradables by definition. As a result, they initially consume relatively more tradables than nontradables, hence the initial real appreciation. In the optimal semi-open economy, the real exchange rate appreciates even more initially. This is because the optimal policy with large ϕ consists in maintaining a relatively

low domestic interest rate in order to alleviate the credit constraint of cash-poor agents. With a large ϕ, cash-poor agents borrow more than cash-rich agents save, so higher overall transfers can be achieved by decreasing the interest rate than by increasing it. This implies that the central bank accumulate fewer reserves than in the open economy, which stimulates the consumption of tradables and appreciates further the currency.

As mentioned in the Introduction, the literature has highlighted the role of pecuniary externalities in order to justify the use of capital controls or, equivalently, of real exchange rate manipulation. This motive is present in our model. It is represented by the first term in the second line of Equation (22). It reflects the will of the central bank to appreciate the real exchange rate in order to inflate the value of the collateral and relax the constraint. In order to assess the role of this effect, we distinguish between the share of tradable goods that can be used as collateral ϕ^T from the share of of nontradable goods that can be used as collateral ϕ^N (e.g., as in Bianchi, 2011), i.e.,

$$r_{t+2}L_{t+2} \leq \phi^N Y_{t+2}^N + \phi^T p_{t+2} Y_{t+2}^N. \tag{23}$$

The pecuniary externality arises only through ϕ^T. We therefore set ϕ^T to zero and set $\phi^N = 0.3(1 + \kappa)/\kappa$ so that agents face the same "average" credit constraint as in the case with larger ϕ, represented in the upper-right panel of Figure 3. The results are represented in the middle-right panel. The dynamics of the real exchange rate is exactly the same as in the case where ϕ^T and ϕ^N are equal, which shows that the pecuniary externality motive is dominated by the other motives for exchange rate manipulation.

Finally, we consider a case with smaller income variability with $a = 0.8$ in the lower-right panel. The dynamics resembles qualitatively that with a larger ϕ. First, better consumption smoothing generates a real appreciation followed by a depreciation. Second, when a is large, agents revenues do not fluctuate much and they thus need to save less than with $a = 0$. As a result, agents borrow more than they save, so it is optimal to decrease the domestic interest rate. This is achieved through a lower foreign asset accumulation and a stronger real initial appreciation in the optimal semi-open economy than in the open economy.

6   Conclusions

TBA

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A Proof of Proposition 1

TO BE ADDED

B First-order condition with respect to B^*

TO BE ADDED

C Derivation of equation (21)

TO BE ADDED

C.1 Derivation of equation (22)

By definition, J^* is the left-hand side of Equation (20), evaluated at $r_{t+1} = r^* = 1/\beta$, so we have:

$$J^*_{t+1} = -(\gamma^G_t - \gamma_{t+1}) + \frac{\Lambda_{t+1}}{2} \tag{24}$$

Consider the first-order conditions with respect to c^{AT}_t:

$$\frac{1+\kappa}{c^{AT}_t} - (1+\kappa)\gamma^A_t - \gamma^G_t - \kappa\gamma^N_t - \frac{\kappa^A_t}{(c^{AT}_t)^2} = 0$$

and with respect to c^{LT}_t:

$$\frac{1+\kappa}{c^{AT}_t} - (1+\kappa)\gamma^A_t - \gamma^G_t - \kappa\gamma^N_t + \frac{\kappa^A_t}{\beta(c^{LT}_t)^2} = 0$$

Subtracting one equation from the other, and using the fact that $c^{AT}_t = c^{LT}_t$, we

obtain:

$$\gamma_t^G - \gamma_{t+1}^G = (1+\kappa)(\gamma_{t+1}^L - \gamma_t^A) + \kappa(\gamma_{t+1}^N - \gamma_t^N) - \frac{1+\beta}{\beta}\frac{\kappa_t^A}{(c_t^{AT})^2} \tag{25}$$

We first determine $\gamma_{t+1}^L - \gamma_t^A$. Consider the respective first-order conditions with respect to A_{t+1} and L_{t+1}, evaluated at $r_{t+1} = r^* = 1/\beta$: $\gamma_t^A + \gamma_t^B = \gamma_{t+1}^L$ and $\gamma_t^L + \gamma_t^B = \gamma_{t+1}^A + \Lambda_{t+1}$. Subtracting one equation from the other, we obtain:

$$\gamma_t^L - \gamma_t^A = \gamma_{t+1}^A - \gamma_{t+1}^L + \Lambda_{t+1} \tag{26}$$

Notice that the first-order conditions with respect to τ_t gives $\gamma_t^L = -\gamma_t^A$. Replacing in Equation (26), we obtain:

$$\gamma_t^L = -\gamma_{t+1}^L + \frac{\Lambda_{t+1}}{2} \tag{27}$$

which gives us $\gamma_{t+1}^L - \gamma_t^A$:

$$\gamma_{t+1}^L - \gamma_t^A = \frac{\Lambda_{t+1}}{2}$$

Second, we obtain γ_t^N using the first-order condition with respect to p_t:

$$\gamma_t^N = \frac{2}{c_t^{AT} + c_t^{LT}} - \frac{\phi\Lambda_t}{1+a}$$

Finally, we obtain $\kappa_t^A/(c_t^{AT})^2$ from the first-order condition with respect to r_{t+1}, using $c_t^{AT} = c_t^{LT}$:

$$\frac{\kappa_t^A}{(c_t^{AT})^2} = \frac{1}{c_t^{AT}}\left[-\gamma_{t+1}^A L_{t+1} + \gamma_{t+1}^L - \Lambda_{t+1}L_{t+1}\right]$$

Iterating Equation (27) forward, we get:

$$\gamma_{t+1}^{L} = \frac{1}{2} \sum_{i=0}^{\infty} (\Lambda_{t+2i+2} - \Lambda_{t+2i+3})$$

Replacing γ_{t+1}^{L} and $\gamma_{t+1}^{A} = -\gamma_{t+1}^{L}$ in $\kappa_t^A/(c_t^{AT})^2$ and rearranging, we obtain:

$$\frac{\kappa_t^A}{(c_t^{AT})^2} = \frac{1}{c_t^{AT}} \left[\left(\sum_{i=1}^{\infty} \Lambda_{t+2i} \right) A_{t+1} - \left(\sum_{i=0}^{\infty} \Lambda_{t+1+2i} \right) L_{t+1} - \frac{1}{2} \left(\sum_{i=1}^{\infty} \Lambda_{t+1+i} \right) (A_{t+1} - L_{t+1}) \right]$$

After replacing the different terms in Equation (25) and replacing $\gamma_t^G - \gamma_{t+1}^G$ in Equation (24), we obtain Equation (22).
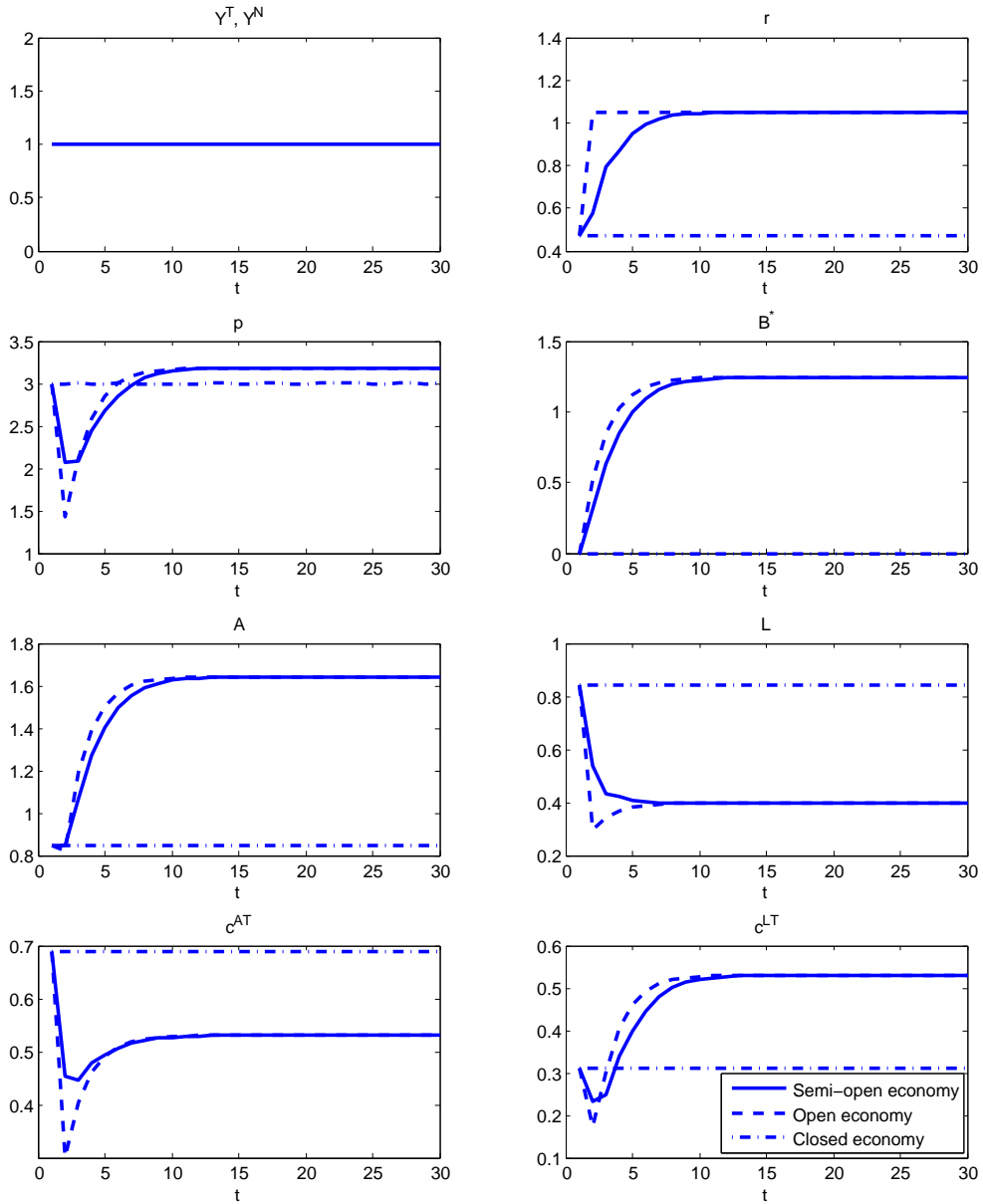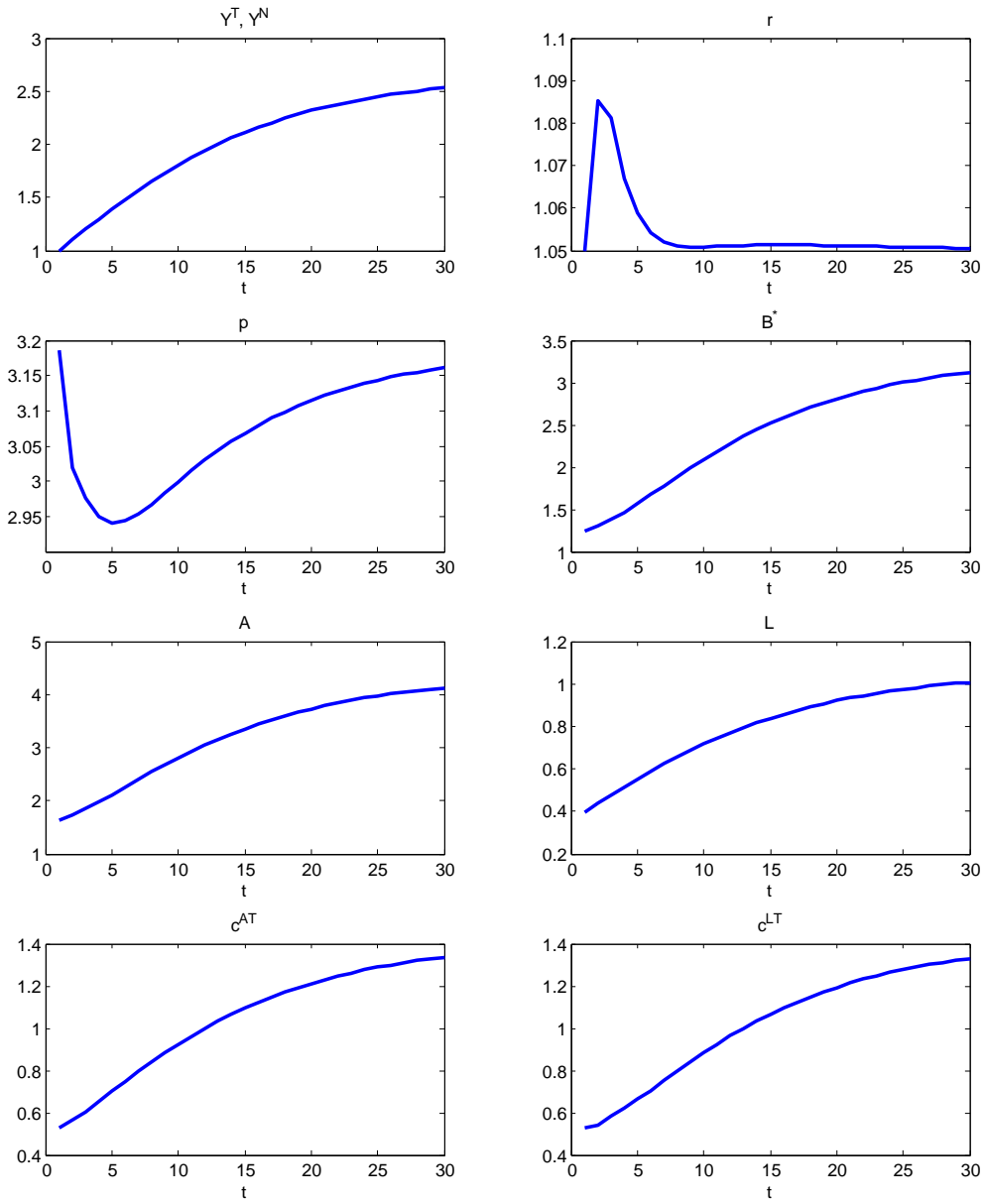
Figure 1: Optimal policy in a closed economy
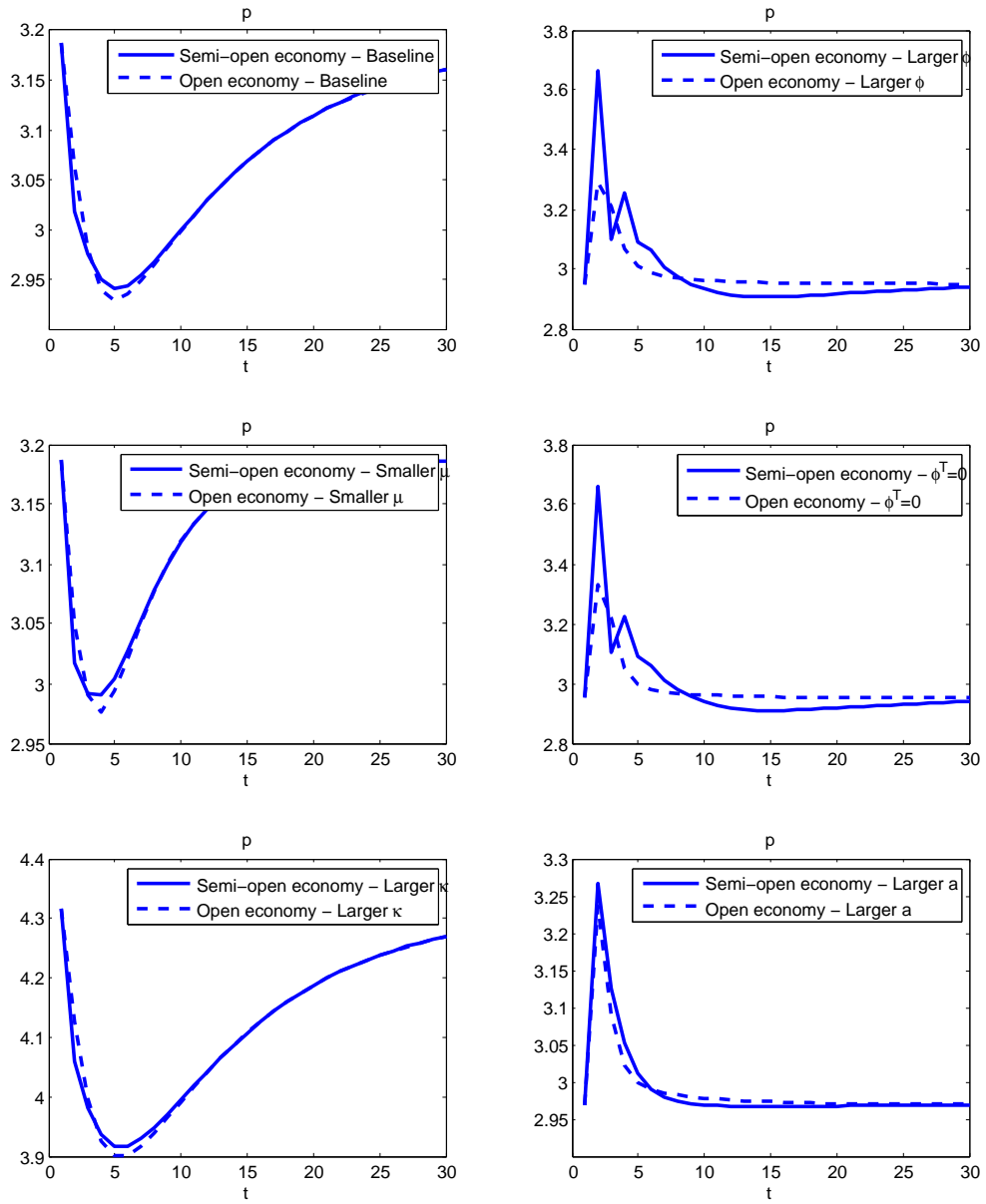
Figure 2: Optimal policy in a catching-up economy - Baseline

Figure 3: Sensitivity analysis