Global Debt-at-Risk

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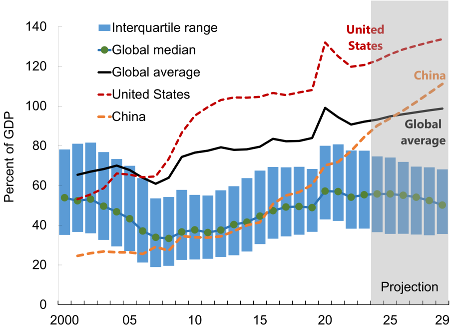
Global public debt is elevated and could be worse than projected

- High debt reduces fiscal space and raises the risk of sovereign stress.
- Essential to examine the risks around the debt outlook.

Government Debt-to-GDP Ratio, 2000-29
(Percent of GDP)



Novel "debt-at-risk" (DaR) framework to quantify the full distribution of risks around debt projections.

Builds on the "growth-at-risk" methodology (Adrian, Boyarchenko, and Giannone 2019):

- Panel quantile regressions for a sample of 74 countries to construct predicted quantiles of future debt at a forecast horizon of one to five years.
- Estimates fitted to a skewed t-distribution.
- Densities conditional on multiple variables are combined to a single distribution based on the individual factors' predictive power.

Distinctive advantages of DaR:

- Goes beyond proximate drivers to consider underlying factors—e.g., financial stress.
- Examines their nonlinear effects on the debt distribution.
- Gauges how high debt could rise in an extreme adverse, but plausible, scenario.
- Assesses the relationship between debt-at-risk and fiscal crises.

Preview of results

- Several financial, political, and economic variables shift the entire future debt distribution, with stronger effects at the right tail.
- Global debt-at-risk three years ahead is 115 percent of GDP, nearly 20 ppts above IMF WEO projections.
  - High current debt amplifies the impact of conditioning factors on debt risks.
- Debt-at-risk differs across countries and country income groups.
  - Advanced economies: 134 percent of GDP; Emerging markets: 88 percent.
  - Tighter financial conditions, social unrest, and policy uncertainty are more strongly associated with debt risks in emerging markets.
- Debt-at-risk predicts fiscal crises; outperforms other common indicators.

Data

Data structure:

- A panel structure: 74 advanced and emerging and developing economies from 1980-2023 (>90 percent of global debt).

Key variables:

- **Financial variables:** Financial Conditions Index (IMF), Financial Stress Index (Ahir et al. 2023), spreads, World Uncertainty Index (Ahir, Bloom, and Furceri 2022).
- **Political variables:** Reported Social Unrest Index (Barrett et al. 2022).
- **Economic variables:** Debt-to-GDP, primary balance, real GDP growth, inflation (IMF projections).

Countries

Quantile regression specification

Estimate the following location-scale model (Machado and Santos Silva 2019):

$$d_{i,t+h} = \alpha_i + X'_{i,t}\beta + (\delta_i + X'_{i,t}\gamma)\varepsilon_{i,t+h} \tag{1}$$

where:

- $d_{i,t+h}$: h year-ahead debt-to-GDP ratio (h :1 to 5 years), of country i in year t.
- α_i and δ_i: Country fixed effects.
- $X_{i,t}$: Vector of predictors (initial debt included in all specifications).
- β: Links the location of the predictive density to predictors.
- γ: Scale parameter for error term $\varepsilon_{i,t+h}$.

The τ-th quantile of future debt, $Q_d(\tau)$ is given by:

$$Q_{d_{i,t+h}}(\tau|X_{i,t}) = (\alpha_i + \delta_i q(\tau)) + X'_{i,t}\beta + X'_{i,t}\gamma q(\tau) \tag{2}$$

where $q(\tau) = F_{\varepsilon}^{-1}(\tau)$

Other Specifications

Fitting and combining densities

Fitting densities: Details

- Re-center quantiles such that predicted median conditional on initial debt only matches IMF 2024-28 projection.
- Fit predicted quantiles to a skewed t-distribution (Azzalini and Capitanio 2003).

Combining densities:

- For a particular country, year, and forecast horizon, we pool densities using a weighted sum of the densities based on individual predictors m:

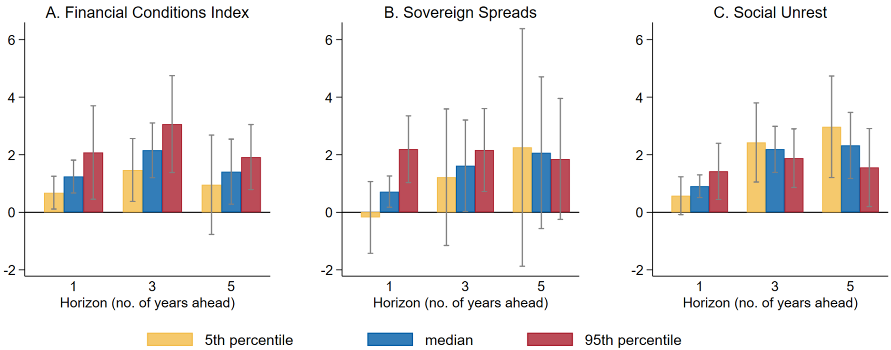
$$\hat{f}_{i,t+h}^{poooled}(d) = \sum_m \eta_{i,h}^m \hat{f}_{i,t+h}^m(d) \tag{3}$$

- Weights $\eta_{i,h}^m$ sum to one and maximize out-of-sample predictive accuracy of the combined distribution (Crump et al. 2023). Details

Aggregation: Quantiles aggregated to global/group level using GDP weights Details

Adverse financial and political developments are associated with higher debt-at-risk

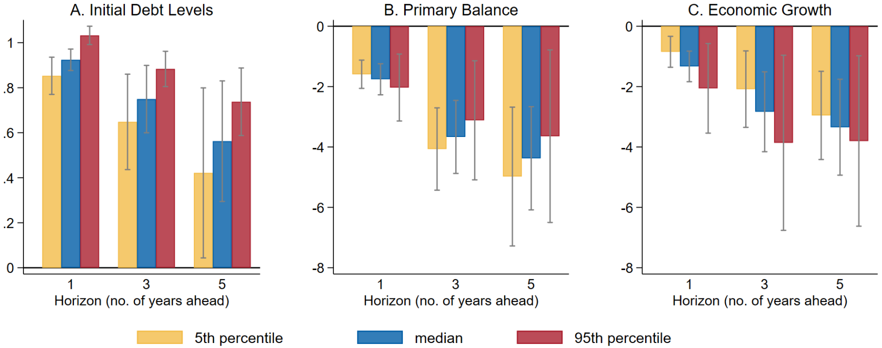
Quantile Regression Coefficients: Forward Debt-to-GDP Ratio and Financial and Political Variables



The figure shows the estimated coefficients for 5th, 50th, and 95th percentiles based on Equation (1). Bars denote estimated coefficients. All variables are standardized to have a mean of zero and a standard deviation of one to ensure comparability across coefficients. The whisker in each bar shows the 90 percent confidence interval for the estimated coefficient.

Economic factors have persistent and asymmetric effects on the debt distribution

Quantile Regression Coefficients: Forward Debt-to-GDP Ratio and Economic Variables

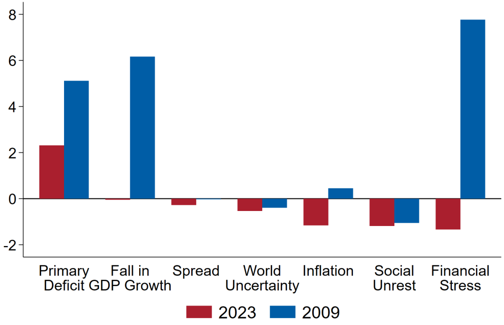


The figure shows the estimated coefficients for 5th, 50th, and 95th percentiles based on Equation (1). Bars denote estimated coefficients. All variables except for initial debt are standardized to have a mean of zero and a standard deviation of one to ensure comparability across coefficients. The whisker in each bar shows the 90 percent confidence interval for the estimated coefficient.

The impact of conditioning factors on debt risks varies over time...

- Primary deficit is the largest driver of global debt risks now, compared to financial stress during the global financial crisis.

Variables Contributing to Three-Year Ahead Global Debt-at-Risk: 2023
(Percent of GDP)



The figure plots the difference between the predicted 95th quantile of three-year-ahead global debt-to-GDP conditional on the variables displayed on the horizontal axis (and initial debt) relative to the 95th quantile conditional on initial debt only.

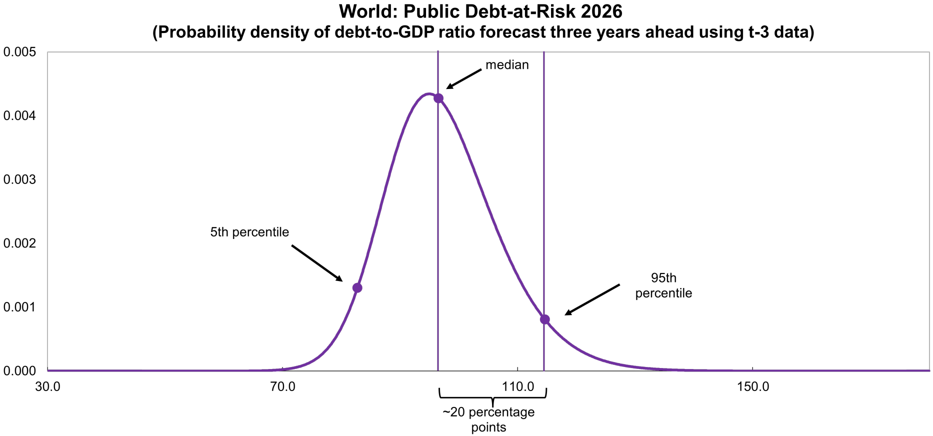
... and across countries

Weights Used to Combine Distributions

Forecast Horizon (Years)	Initial Debt	Financial Stress Index	Spread	World Uncertainty Index	Social Unrest Index	Primary Balance	GDP Growth	Inflation
United States								
1	-	0.62	-	-	-	0.38	-	-
2	-	0.35	-	-	-	0.65	-	-
3	-	0.22	-	-	-	0.78	-	-
4	-	0.23	-	-	-	0.77	-	-
5	-	0.58	-	-	-	0.42	-	-
Brazil								
1	-	-	-	0.86	0.06	-	0.08	-
2	-	-	-	-	1.00	-	-	-
3	0.01	-	0.24	0.02	0.56	0.01	0.14	0.01
4	0.01	-	-	-	0.44	0.04	0.51	-
5	0.01	-	-	-	0.76	0.02	0.06	0.14

The table displays the weights used to combine the conditional distributions based on each conditioning variable into a single distribution for the United States and Brazil. The procedure used to compute the weights follows Equation (7).

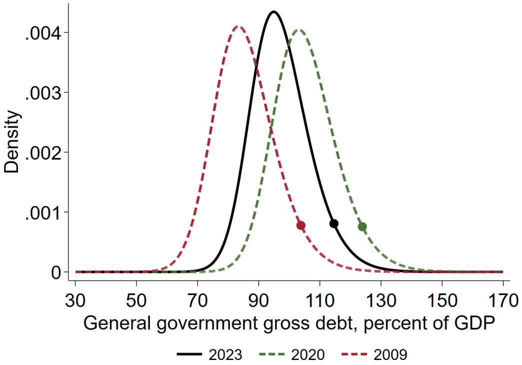
Global debt-at-risk for 2026 is estimated at 115 percent of GDP, nearly 20 ppts higher than projections



The probability density functions are estimated using panel quantile regressions of the debt-to-GDP ratio on various political, economic, and financial variables. The global sample comprises 74 countries—accounting for more than 90 percent of global debt—for which data on the conditioning variables are available. The quantile estimates are fitted to a skewed t distribution for every year in the sample. Dots indicate the predicted 5th, 50th (median), and 95th quantile of the debt-to-GDP ratio.

The predicted global debt distribution is as right skewed as it was after the financial crisis

Global Debt-at-Risk and Its Evolution
(Probability density of three-year-ahead government debt-to-GDP ratio)



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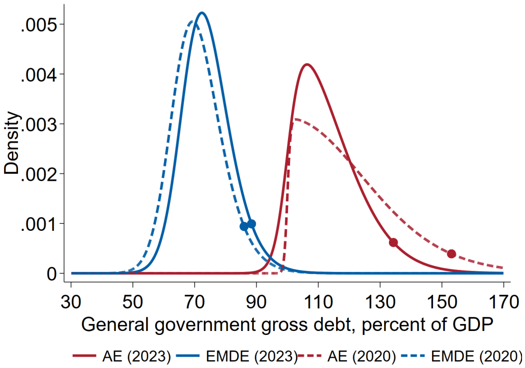
Debt-at-risk varies significantly across countries and country groups

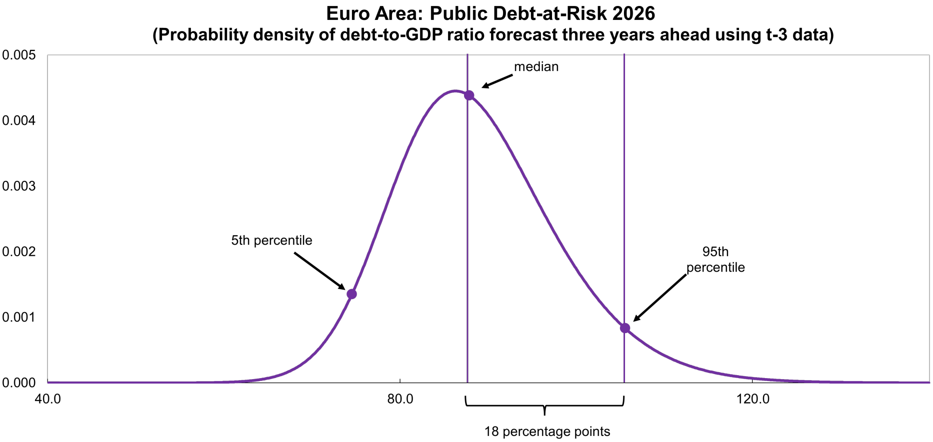
Debt-at-Risk across Income Groups
(Probability density of three-year-ahead government debt-to-GDP ratio)



The probability density functions are estimated using panel quantile regressions of the debt-to-GDP ratio on various political, economic, and financial variables. The quantile estimates are fitted to a skewed t distribution for every year in the sample. Dots indicate the predicted 95th quantile of the debt-to-GDP ratio for each country group.

By Region

Euro-Area debt-at-risk for 2026 is estimated at 106 percent of GDP, 18 percentage points higher than projections



The probability density functions are estimated using panel quantile regressions of the debt-to-GDP ratio on various political, economic, and financial variables. The global sample comprises 74 countries—accounting for more than 90 percent of global debt—for which data on the conditioning variables are available. The quantile estimates are fitted to a skewed t distribution for every year in the sample. Dots indicate the predicted 5th, 50th (median), and 95th quantile of the debt-to-GDP ratio.

Debt-at-Risk and fiscal crises

1. Does the debt-at-risk measure help predict fiscal crises?
2. If so, how well do they perform relative to other economic variables?

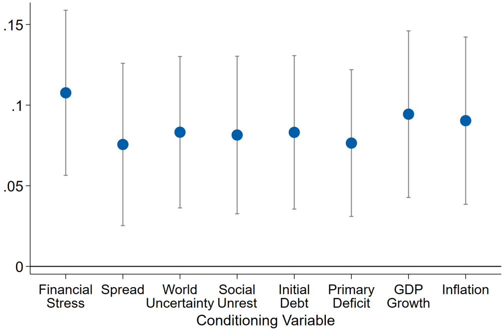
To address these, we:

- Construct a binary fiscal crisis variable (Moreno Badia et al. 2022). Details
- Correlate crisis variable with debt-at-risk measure:
  - Logit model.
  - Random forest machine learning model.

1. Does the debt-at-risk measure help predict fiscal crises?

Debt-at-Risk across all predictive models correlates strongly and positively with a future fiscal crisis.

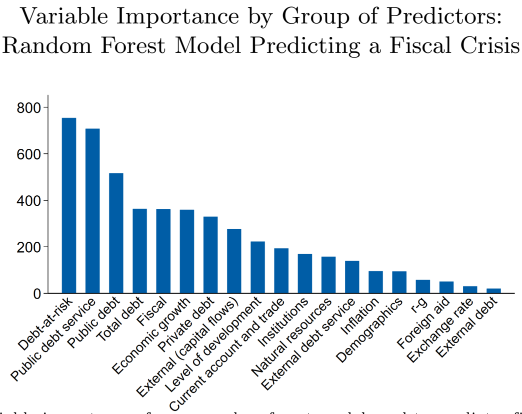
Logistic Regression Coefficients:
Fiscal Crisis vs. One-Year Ahead Debt-at-Risk



The figure shows estimated coefficients from a panel logit regression of a fiscal crisis indicator against debt-at-risk. Each point denotes the coefficient from a separate regression. The independent variable is the difference between the predicted 95th quantile of one-year-ahead debt-to-GDP and the 50th quantile conditional on the variables displayed on the horizontal axis. Whiskers show the 90 percent confidence intervals.

2. How well does the measure perform relative to other economic variables?

Debt-at-risk is the most strongest metric in predicting a fiscal crisis.



Without DaR

The figure displays (grouped) variable importances from a random forest model used to predict a fiscal crisis. Higher values indicate that a variable has a higher predictive power. Variable importance is calculated using an in-built out of bag permuted predictor importance function in R using a random forest with the variables selected by Boruta.

Conclusion and policy implications

Conclusion:

- Debt risks are elevated and titled to the upside.
- Global debt-at-risk in 2026 is estimated at 115 percent of GDP, with significant heterogeneity across countries.
- Debt-at-risk is the most robust predictor of fiscal crises.

Policy implications:

- Policymakers can quantify the size of debt risks and compare risks over time and across countries.
- Debt-at-risk could be a useful early-warning tool for fiscal crises.

Additional Slides

List of Countries Used in Debt-at-Risk Analysis

Advanced Economies	**Emerging Market and Middle Income Economies**	**Low Income Developing Countries**
Australia	Argentina	Bangladesh
Austria	Armenia	Côte d'Ivoire
Belgium	Botswana	Kenya
Canada	Brazil	Nigeria
Croatia	Bulgaria	Tanzania
Cyprus	Chile	Uganda
Czech Republic	China	Zambia
Denmark	Colombia	
Estonia	Ecuador	
Finland	Egypt	
France	Hungary	
Germany	India	
Greece	Indonesia	
Hong Kong SAR	Kazakhstan	
Iceland	Malaysia	
Ireland	Mexico	
Israel	Morocco	
Italy	Namibia	
Japan	Pakistan	
Korea	Peru	
Latvia	Philippines	
Lithuania	Qatar	
Luxembourg	Romania	
Malta	Russia	
Netherlands	South Africa	
New Zealand	Sri Lanka	
Norway	Thailand	
Portugal	Tunisia	
Singapore	Türkiye	
Slovak Republic	Vietnam	
Slovenia		
Spain		
Sweden		
Switzerland		
Taiwan Province of China		
United Kingdom		
United States		

The table displays the countries included in the sample of 74 economies used for the debt-at-risk analysis. Country classification is per the IMF's WEO database.

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Quantile regression specifications: heterogeneity by initial debt and country group

Initial debt: Include nonlinear interactions with contemporaneous debt:

$$X'_{it}\beta = \sum_{k=1}^{4} \beta_{1,k} x_{i,t} \times \mathbf{1}\{Q(d_{i,t}) = k\} + \sum_{k=1}^{4} \beta_{2,k} d_{i,t} \times \mathbf{1}\{Q(d_{i,t}) = k\} \tag{4}$$

where $\mathbf{1}\{Q(d_{i,t}) = k\}$ equals one if the quartile of the debt-to-GDP ratio equals k; $k \in \{1, 2, 3, 4\}$.

Country group: Include interactions with country income group:

$$X'_{it}\beta = \sum_{j=\{AE,EMDE\}} \beta_{1,j} x_{i,t} \times \mathbf{1}\{country\, i \in j\} + \sum_{j=\{AE,EMDE\}} \beta_{2,j} d_{i,t} \times \mathbf{1}\{country\, i \in j\} \tag{5}$$

where $\mathbf{1}\{country\, i \in j\}$ equals one if a country is classified as an advanced economy or emerging market and developing economy, respectively. Back

Fitting densities to a skewed t-distribution

- Distribution depends on four parameters: the location μ, scale σ, fatness ν, and the shape α.
- Choose parameters to fit predicted quantiles to the quantile function of the distribution:

$$\{\hat{\mu}^m_{i,t+h}, \hat{\sigma}^m_{i,t+h}, \hat{\alpha}^m_{i,t+h}, \hat{\nu}^m_{i,t+h}\} = \underset{\mu,\sigma,\alpha,\nu}{\operatorname{argmin}} \sum_{\tau} \left(\hat{Q}^m_{d_{i,t+h}}(\tau) - F^{-1}(\tau; \mu, \sigma, \alpha, \nu) \right)^2 \tag{6}$$

where:

- $\hat{Q}^m_{d_{i,t+h}}(\tau)$ is predicted quantile from quantile regression (2).
- $F^{-1}(\tau; \mu, \sigma, \alpha, \nu)$ is the quantile function of the skewed t-distribution.
- $\tau \in \{5, 25, 75, 95\}$

Back

Weighting methodology to combine conditional distributions

Two steps:

- Compute out-of-sample predictive densities conditional on each explanatory variable m using data from the *prior* 20 years (from 2005 onwards).
  - E.g., $\hat{p}^m_{2007|2005}(d)$ is density for 2007 conditional on information from 1986 to 2005.
- Weights are the values (positive and summing to one) that maximize these probability scores across all years:

$$\eta^1_{i,h}, \dots, \eta^M_{i,h} = \text{argmax} \sum_{t=2005+h}^{2023} \sum_{m=1}^{M} \eta^m_{i,h} \hat{p}^m_{T+h|T}(d) \tag{7}$$

such that $\eta_{i,h} > 0; \sum_{m=1}^{M} \eta^m_{i,h} = 1$

Back

Aggregating to global levels or country groups

- For each model m, approximate the quantile of the global distribution with the weighted average of the country-level quantiles:

$$\hat{Q}^{m}_{d_{global,t+h}}(\tau) = \sum_{i=1}^{I} \omega_{i,t} \hat{Q}^{m}_{d_{i,t+h}}(\tau) \tag{8}$$

- $\omega_{i,t}$ is country i's nominal GDP share.
- Re-center quantiles around WEO projections.
- Fit aggregate quantiles to skewed t-distribution to obtain global density $\hat{f}^{m}_{global,t+h}(d)$
- Pool densities by combining model-specific densities; global weights are the GDP-weighted average of the country-specific weights.

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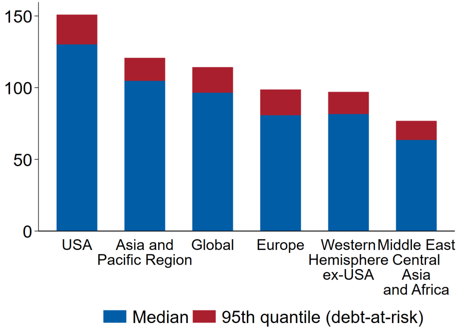
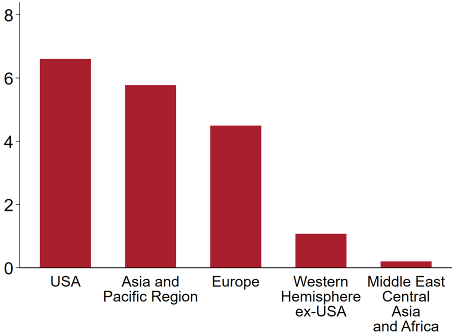
Debt-at-risk by region

Debt-at-Risk by Region
(Percent of GDP)



Regional Contribution to Global Debt-at-Risk
(Percent of GDP)



The left figure plots the three-year-ahead predicted median and 95th quantile debt-to-GDP ratio by region. The right figure plots the difference between the predicted 95th quantile and the (unconditional) predicted median for each region. This difference is then weighted by the region's nominal GDP to create a contribution to global debt-at-risk that aligns with the approach used to create the global quantiles (Equation (8)). For both figures, the regional aggregates only include the countries in the 44 country sample that are used to create the global distribution.

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Defining a fiscal crisis

Fiscal crisis occurs if any one of four criteria are met:

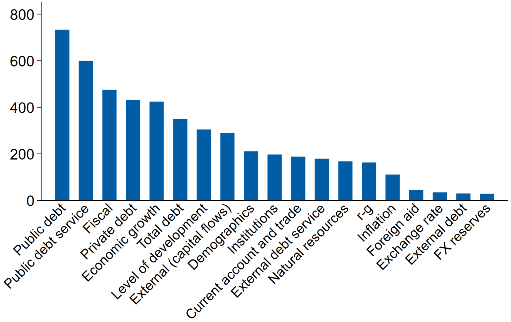
1. A credit event (default, restructuring, or rescheduling).
2. Exceptionally large official financing from the IMF or European Union.
3. Implicit default on domestic debt (high inflation, increase in domestic arrears).
4. Loss of market confidence (spike in spreads, loss of market access).

Back

Variable importances without debt-at-risk variables

Public debt levels and debt service are the most important predictors of fiscal crises, consistent with literature

Variable Importance by Group of Predictors:
Random Forest Model Predicting a Fiscal Crisis (Excluding Debt-at-Risk)



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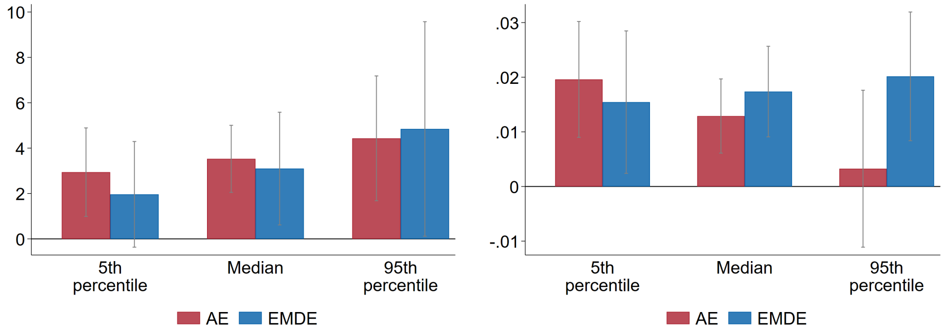
Financial conditions and social unrest are more strongly associated with debt risks in EMDEs

Debt-at-Risk by Country Groups:
Coefficient on Three-Year-Ahead Debt-to-GDP Ratio

A. Financial Conditions Index

B. Social Unrest



The figure shows estimated coefficients for the 5th, 50th, and 95th percentiles based on Equation (5) for the FCI (Panel A) and RSUI (Panel B) for AEs (advanced economies) and EMDEs (emerging market and developing economies). The dependent variable is the three-year-ahead debt-to-GDP ratio. Bars denote estimated coefficients. Whiskers in bars show 90 percent confidence intervals. for estimated coefficients.

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