Is Automating AI Research Enough for a Growth Explosion?

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April 2025

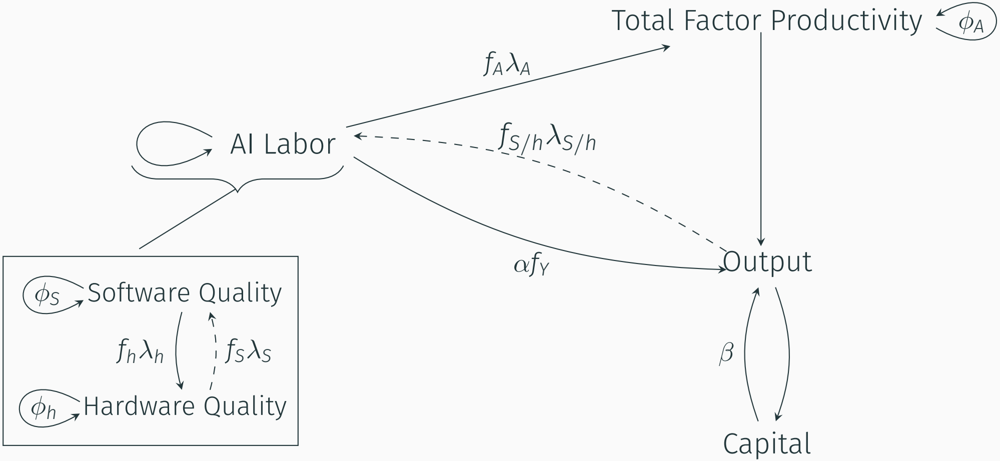
Motivation

> "Perhaps some areas, like robotics, might take longer to figure out by default. And the societal rollout, e.g. in medical or legal professions, could easily be slowed by societal choices or regulation. But **once models can automate AI research itself, that's enough—enough to kick off intense feedback loops—and we could very quickly make further progress, the automated AI engineers themselves solving all the remaining bottlenecks to fully automating everything.** In particular, millions of automated researchers could very plausibly compress a decade of further algorithmic progress into a year or less."

Situational Awareness, Aschenbrenner (2024)

The software-hardware model of AI

The software-hardware model of AI



Roadmap ▸ Lit review

1. Building blocks of the model

2. The software-hardware model

3. Scope of claims

Building blocks of the model

Building blocks of the model

The software-hardware model

Scope of claims

The intelligence explosion

> **“Let an ultraintelligent machine be defined as a machine that can far surpass all the intellectual activities of any man.**
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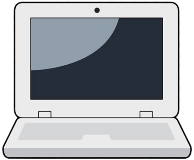

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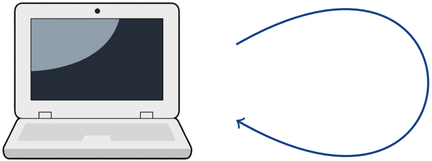

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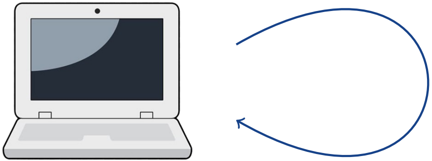

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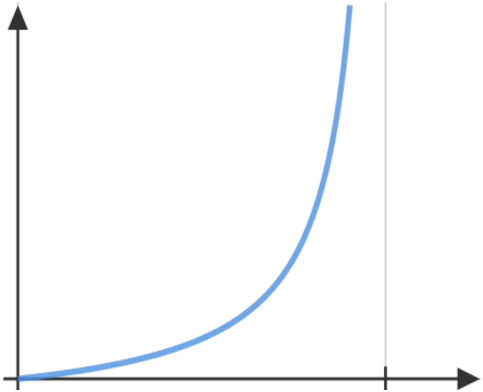
$$\phi > 0$$



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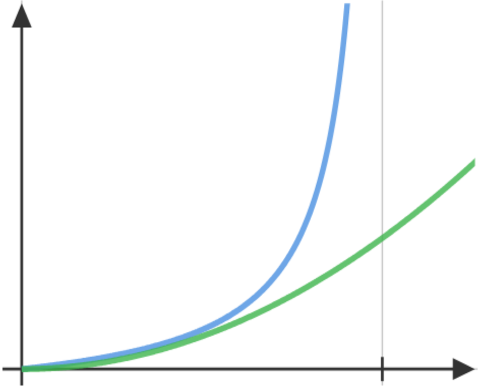
$$\dot{A}_t = A_t^{1+\phi}$$

$$\phi \in (-1, 0)$$



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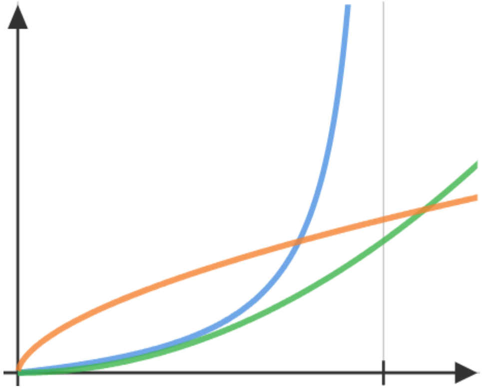

The intelligence explosion? The role of diminishing returns

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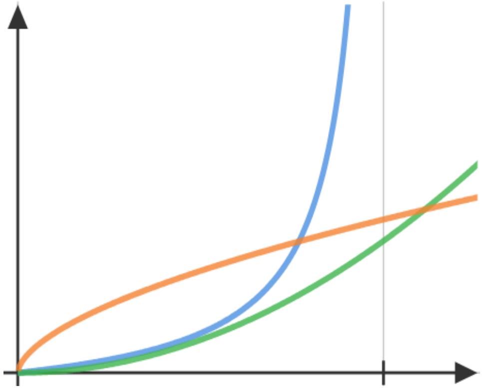

Will an *intelligence* explosion cause an *economic* explosion?

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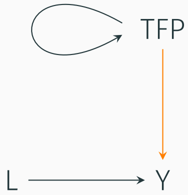

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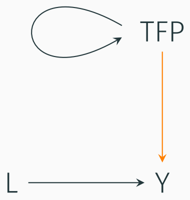
If $\gamma > 0$, any form of intelligence explosion causes the same form of economic explosion



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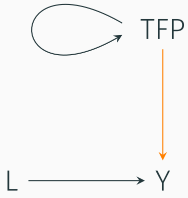

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- **Economic singularity** condition:

$$\phi > 0$$

Other feedback loops matter:

$$\dot{A}_t = A_t^{1+\phi}$$
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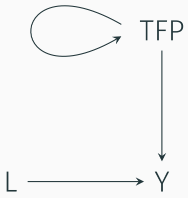

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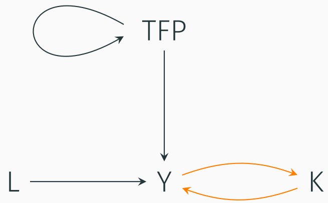

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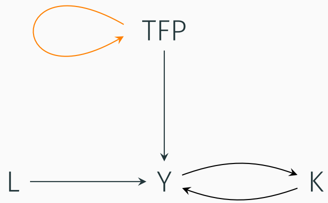

Other feedback loops matter: the role of *accumulable factors*

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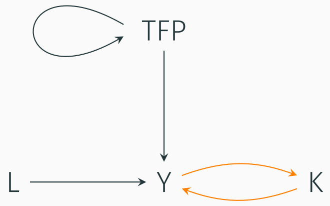

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$$\dot{A}_t = A_t^{1+\phi} (\kappa K_t)^{\lambda}$$

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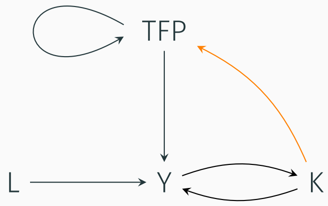

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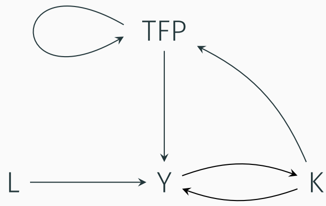

Focusing on accumulable factors:

$$\dot{A}_t = \text{stuff} \cdot A_t^{1+\phi} K_t^{\lambda}$$
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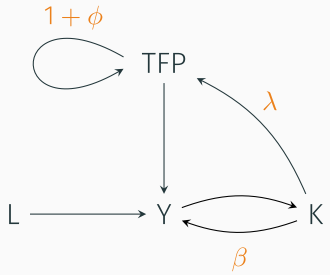

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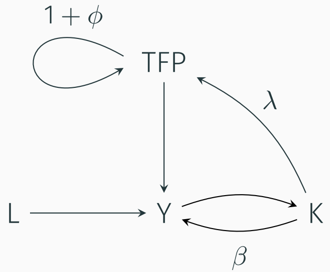

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Proposition (explosive systems).
System explodes in finite time if the exponent matrix, $\begin{bmatrix} 1+\phi & \lambda \\ 1 & \beta \end{bmatrix}$, has an eigenvalue > 1.

Explosion conditions:

$\phi > 0$ or $\beta > 1$

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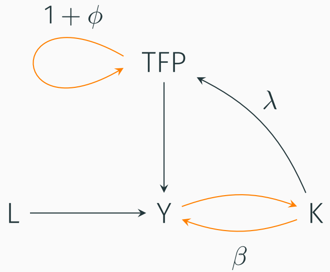

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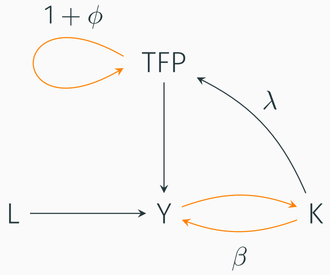

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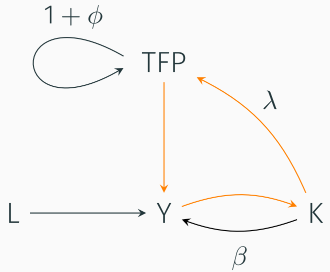

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The canonical semi-endogenous growth model

$$\dot{A}_t = A_t^{1+\phi}(\ell L_t)^{\lambda}(\kappa K_t)^{\lambda}$$

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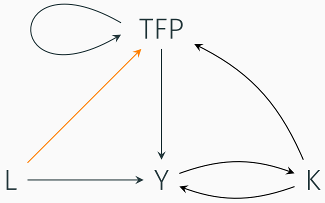

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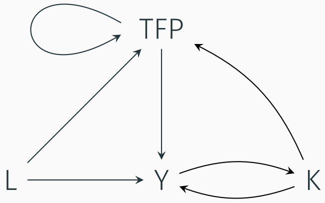

Best guess calibration:

- $\phi = -3.4$ (Bloom et al 2020) — $\phi > 0$ ✗
- $\beta = 0.4$ (capital share in production) — $\beta > 1$ ✗
- $\lambda = 0.1$ (capital share in R&D) — $(1+\phi)+\beta-(1+\phi)\beta+\lambda > 1$ ✗

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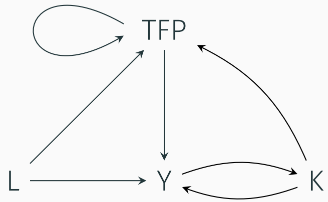

Takeaways:

1. Where are the feedback loops?

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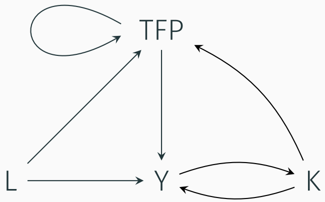

Takeaways:

1. Where are the feedback loops?
2. How strong are the diminishing returns?

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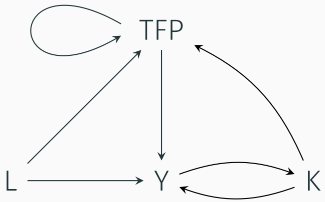

Takeaways:

1. Where are the feedback loops?
2. How strong are the diminishing returns?
3. What are the accumulative factors?

Introducing automation

$$\dot{A}_t = A_t^{1+\phi}(\ell_A L_t)^{\lambda}(\kappa_A K_t)^{\lambda}$$
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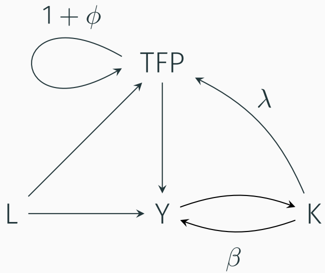

Explosion conditions:

$$\phi > 0$$
$$\beta > 1$$
$$(1+\phi) + \beta - (1+\phi)\beta + \lambda > 1$$

Introducing automation

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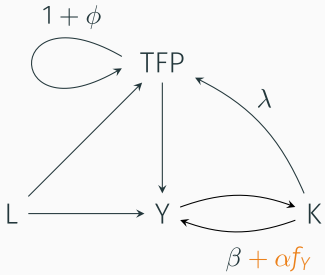

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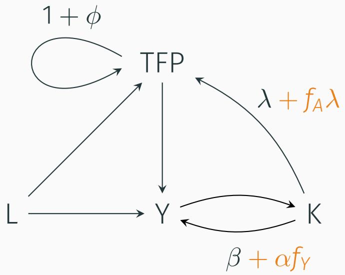

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The software-hardware model

Building blocks of the model

The software-hardware model

Scope of claims

Software-hardware model: overview

Canonical semi-endogenous growth model, plus:

Software-hardware model: overview

Canonical semi-endogenous growth model, plus:

1. Automation of labor with “AI”

Software-hardware model: overview

Canonical semi-endogenous growth model, plus:

1. Automation of labor with “AI”
2. AI = software · hardware · hardware quality

The software-hardware model of AI

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AI substituting for labor:

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- **Hardware:** computer hardware ("compute")

The software-hardware model of AI

AI substituting for labor:

$$\text{AI} \equiv Z = \underbrace{S}_{\text{software}} \cdot \underbrace{C}_{\text{hardware}} = \underbrace{S}_{\text{software}} \cdot \underbrace{c \cdot h}_{\text{hardware}}$$

- **Software:** "algorithmic efficiency"
- **Hardware:** computer hardware ("compute")
  - Hardware quantity: c, "number of computer chips"
  - Hardware quality: h, "how many calculations (FLOPs) per chip"

Software and hardware evolution

Hardware accumulates: just another form of capital

$$C_t = s_C Y_t - \delta_C C_t$$

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$$\dot{C}_t = s_C Y_t - \delta_C C_t$$

Software is like ideas: better software allows for *faster software progress*

$$\dot{S}_t = (\ell_S L_t)^{\lambda_S} S_t^{1+\phi_S}$$

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Software is like ideas: better software allows for *faster software progress*

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Remember ideas production function:

$$\dot{A}_t = (\ell_A L_t)^{\lambda_A} A_t^{1+\phi_A}$$

Software and hardware evolution

Hardware accumulates: just another form of capital

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[a la Greenwood-Hercowitz-Krusell]

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Note: effective labor accumulates

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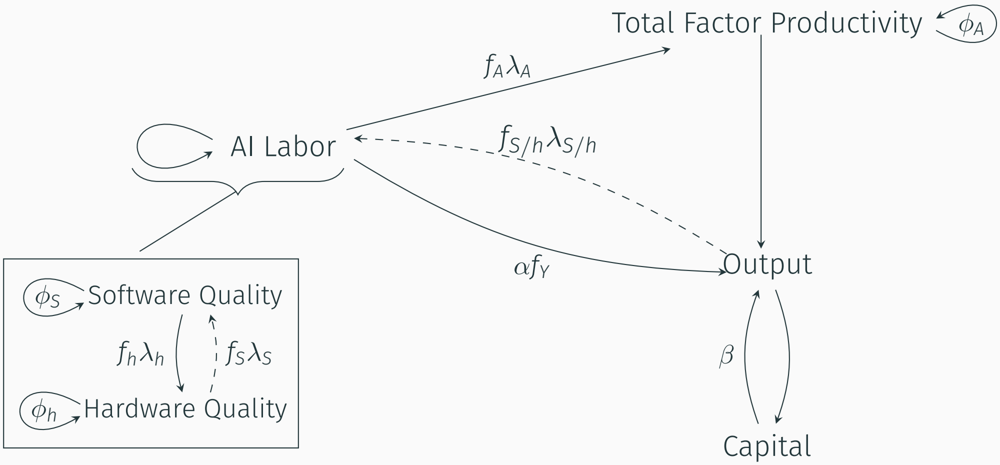
The software-hardware model: diagram



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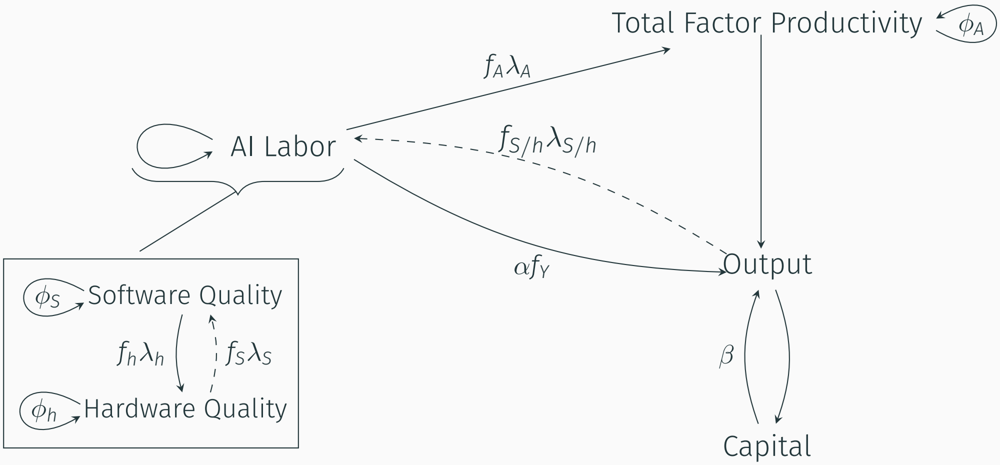

Strength of feedback increasing with all exponents

Explosion condition

Simplify the problem by assuming complete depreciation. Substituting in effective labor expressions and removing non-accumulable factors

$$\dot{S}_t \propto S_t^{f_S\lambda_S\frac{1-\beta}{1-f_Y\alpha-\beta}+1+\phi_S} h_t^{f_S\lambda_S\frac{1-\beta}{1-f_Y\alpha-\beta}} A_t^{\frac{f_S\lambda_S}{1-f_Y\alpha-\beta}}$$

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r **factor:** for $x \in \{A, S, h\}$,

$$r_x \equiv \frac{\lambda_x}{-\phi_x}$$

- Intuition: in canonical model, $g_A = r_A \cdot$ population growth

Calibrating parameters

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Interpretation: Software and hardware have **much lower** diminishing returns to research than the rest of the economy $\implies$ if software/hardware grow as share of economy, large growth effects

Scope of claims

Building blocks of the model

The software-hardware model

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What we do *not* speak to

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5. **More:**
   - Endogenous allocation rules
   - Decentralized allocation: roles of industrial organization + externalities
   - Learning by doing
   - Capital adjustment costs
   - Time to build

Conclusion: “Why wasn’t automating agriculture enough for a growth explosion?”

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3. Bottlenecks or other limits: we do not speak to *all* limits

Thank you!

Appendix

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On bottlenecks

Cobb-Douglas: with $\alpha > 0$

$$Y = L^{\alpha} K^{1-\alpha}$$

Fix L, send $K \to \infty \implies Y \to \infty$.

Potential bottlenecks:

- **Compute** bottlenecking algorithmic progress
- **Algorithmic progress** bottlenecking compute
- **Energy** bottlenecking everything
- **Data** bottlenecking everything

CES with complements: with $\phi < 0$

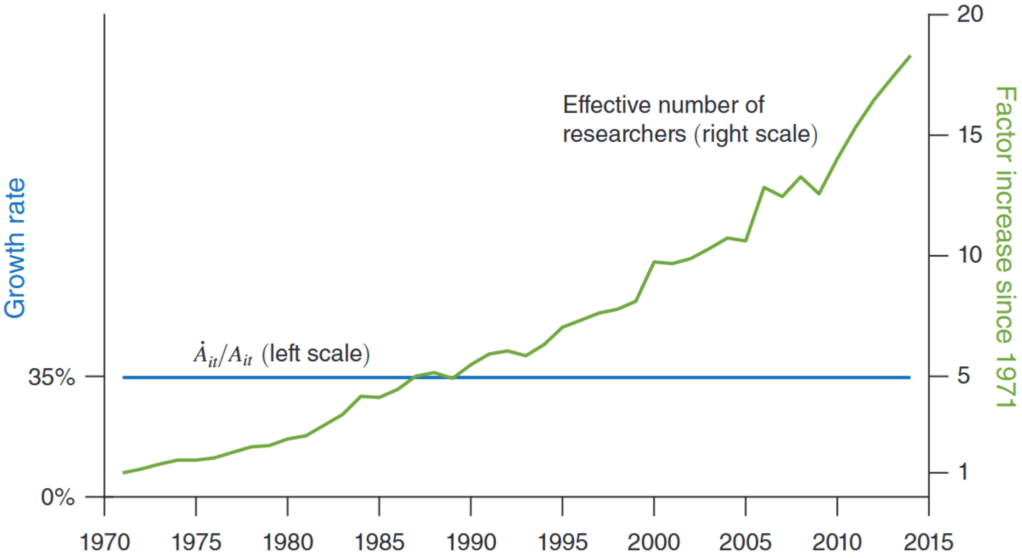
$$Y = \left[L^{\phi} + K^{\phi}\right]^{1/\phi}$$

Fix L, send $K \to \infty \implies Y = L$

Potential reasons to think bottlenecks will be less of an issue:

- 2x efficient algorithims $\implies$ 2x as many experiments
- Aum and Shin (2024): software and labor are substitutes not complements

Could ϕ be falling over time? Doesn't appear to be for Moore's Law



Multisector semi-endogenous growth model

Standard one-sector model:

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Comparative static: Suppose $-\phi_1 > -\phi_2$. Increase σ_2. Obviously $g_A \uparrow$

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